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A COMPARISON OF BUFFER STRIP AND  
NON-BUFFER STRIP JOINT DESIGNS

James Michael Gill

# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



# THESIS

A COMPARISON OF BUFFER STRIP AND NON-BUFFER  
STRIP JOINT DESIGNS

by

James Michael Gill

June 1976

Thesis Advisor:

M.H. Bank II

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A COMPARISON OF BUFFER STRIP AND NON-BUFFER STRIP  
JOINT DESIGNS

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from the

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June 1976

## ABSTRACT

Buffer strip and non-buffer strip bolted wing skin type joints made from NARMCO 5208/T300 graphite-epoxy material were designed, and the excess bearing capacity and weight of these joints were calculated for a wide range of laminate compositions, bolt hole sizes, and number of bolt holes. Design load conditions representative of an advanced fighter type aircraft were chosen. Joint designs were arbitrarily restrained by assumed manufacturing conditions, assumed interface conditions, and imposed laminate composition restrictions. Charts were prepared from which relative joint efficiencies could be determined but no attempt was made to analyze the effect of the arbitrary design restrictions. The advantages and penalties for buffer strip design were discussed and recommendations for future studies were made.

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## LIST OF SYMBOLS

- AD = overall buffer strip joint width (in.)
- $A_n$  = constant of proportionality between the bolt load P and the portion of P reacted each of the primary strips of a buffer strip joint
- B = excess bearing capacity
- D = bolt hole diameter (in.)
- $E_x$  = tensile modulus (lbf./in.<sup>2</sup>)
- $E_{x_1}$  = tensile modulus of the primary strips in a buffer strip joint (lbf./in.<sup>2</sup>)
- $E_{x_2}$  = tensile modulus of the buffer strip material in a buffer strip joint (lbf./in.<sup>2</sup>)
- $F_{BP}$  = bypass force (lbf.)
- $F_S$  = shear load passing  $P_1$  from a buffer strip to a primary strip (lbf.)
- $F_r$  = reaction force (lbf.)
- $f(\frac{a}{r})_Q^i$  = effective isotropic stress concentration factor at location i for load condition  $\alpha$
- $f_s$  = shear stress between the buffer and primary strips of a buffer strip joint (lbf./in.<sup>2</sup>)
- i = indicator of exact position on the hole
- L = tensile load (lbf.)
- $\ell$  D = side length (in.)
- M = number of rows of bolts in a buffer strip joint
- m = reaction moment (in.-lbf.)
- N = number of rows of bolts in a non-buffer strip joint
- $N_x$  = tensile load (lbf./in.)



$N_{xy}$  = shear load (lbf./in.)  
 $P$  = tensile bolt load (lbf.)  
 $P_s$  = shear bolt load (lbf.)  
 $P_1$  = portion of  $P$  reacted in each of the primary strips  
of a buffer strip joint (lbf.)  
 $P_2$  = portion of  $P$  reacted in the buffer strip material of  
a buffer strip joint (lbf.)  
 $R$  = resultant bolt load (lbf.)  
 $S_1$  = representative applied stress (lbf./in.<sup>2</sup>)  
 $S_2$  = representative failure stress (lbf./in.<sup>2</sup>)  
 $t$  = plate thickness (in.)  
 $t^*$  = effective plate thickness (in.)  
 $t_{\pm 45}$  = total thickness of  $\pm 45^\circ$  laminae (in.)  
 $W_B$  = width of buffer strip material (in.)  
 $W_1$  = half-width of primary laminate material in a buffer  
strip joint (in.)  
 $Z$  = percentage of zero degree plies in the primary  
strips of a buffer strip joint  
 $\alpha$  = subscript denoting applied load conditions as  
follows:  
= bx      bearing in x direction  
= by      bearing in y direction  
= tx      tension in x direction  
= ty      tension in y direction  
= xy      shear  
 $\epsilon$  = strain (in./in.)  
 $\lambda_{\alpha}^i$  = finite width correction factor at location  $i$  for

applied load condition  $\alpha$

$\eta$  = indicator of position in a joint

$\sigma_{BR}$  = resultant bolt bearing stress (lbf./in.<sup>2</sup>)

$\sigma^i$  = net tangential stress at location  $i$

$\sigma_{\alpha}$  = applied stress for load condition  $\alpha$

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## I. INTRODUCTION

### A. BACKGROUND

Aerospace structural design requirements have often been characterized by a demand for high strength and low weight. In many cases, the design possibilities have been limited by the available manufacturing technology and the acceptable manufacturing costs. The most successful designs were usually those which met all these constraints most efficiently.

The demand for high structural efficiency led to the development of advanced laminated composites which have higher strength-to-weight ratios and better fatigue properties than conventional structural materials (Ref. 1). As discussed in Ref. 2, it has also been possible to build laminated composite materials with higher modulus values than those characteristic of conventional materials. Accurate methods for tailoring the properties of such materials have evolved and such tailoring has become accepted design practice (Refs. 3,4,5). Waddoups, in Ref. 6, explained that significant gains in structural efficiency have been demonstrated by doing no more than substituting such a tailored composite for a conventional material keeping the geometry and mating interfaces the same.

These gains were realized because the composite materials used were less dense than the conventional

structural materials which they replaced. These material substitutions did not exploit the high strength and high modulus values achievable with advanced laminated composites.

## B. OBJECTIVES

This study was intended to demonstrate the structural efficiencies which could be achieved by designs which took into account some of the high strength and high modulus properties of advanced composites. It was decided to demonstrate these efficiencies by analyzing the behavior of a plate of laminated material in the region of a bolted joint. This situation corresponded to that of a wing skin attached by a bolted fitting to the fuselage of an aircraft.

It was assumed that the wing skin would be attached to the aircraft fuselage through an aluminum alloy fitting. A maximum interbolt strain level,  $\epsilon$ , of 3000 micro-inches per inch and an inter-bolt spacing of four hole diameters were taken as representative of such fittings.

Design conditions representative of an advanced fighter type aircraft were chosen. The joint was to carry a tensile load,  $N_x$ , of 20,000 lbf. per inch of chord. It was assumed that the joint fittings would be covered by an aerodynamic fairing, and a maximum joint length of ten inches was allowed.

To keep the joint manufacture as simple as possible, it was decided that the skin thickness would vary linearly in the joint and that for any given joint design, all holes

would be of the same diameter. To simplify the analysis it was decided to limit the candidate materials to balanced design laminates composed of zero and  $\pm 45$  degree plies of uniform thickness and material composition. No attempt was made to judge the effect of these restrictions on design efficiency.

The interface requirements determined by the aluminum fitting, and the geometric requirements to satisfy manufacturing simplicity, limited the possible joint designs and prevented full utilization of the high strength and high modulus properties available in the selected materials. It was felt, however, that even under these restrictions it would still be possible to demonstrate significant structural efficiencies by properly tailoring the laminates used in the wing skin.

### C. RELIABILITY CONSIDERATIONS

Aerospace structural designs have had to meet difficult requirements for reliability. These designs have had to be sufficiently strong to carry the required loads and light enough to work in the aerospace environment. In addition to these requirements, critical components of aerospace structures have been expected to demonstrate that they are "fail-safe" and, in military applications at least, to some degree battle damage tolerant.

Early composite materials were judged inadequate for aerospace applications because they could not meet these reliability conditions. At first, quantity production of

advanced laminates was impossible because the required quality control technology did not exist. Wide batch-to-batch variation of the properties of these early materials justified only low confidence levels in their structural reliability. Although the early advanced composites were considered unreliable, materials research continued, driven by the anticipated structural efficiencies such materials could make possible. In 1973, Kaminski reported that NARMCO 5208/T300 graphite-epoxy laminates could meet the anticipated requirements for high strength, high modulus, and low weight and could be manufactured reliably and delivered with minimal batch-to-batch variation of material properties (Ref. 2). This was chosen as the structural material to be used in this study.

The requirement for battle damage tolerance and fail-safe design is met by a variety of methods in designs with conventional materials. These methods include built in high excess load bearing capacity, alternate load paths, and various crack stoppage stress relief devices. All of these techniques can be applied to design with composite materials. In addition to these techniques there is a "buffer strip" material fabrication technique applicable to laminated composites which provides an integral crack stoppage capability. Kaminski and Eisenmann explained this technique in Ref. 7.

In buffer strip design, integral "buffer strips" of low modulus, high fracture toughness material are included

in the laminate. These strips are spaced so that cracks originating in the high modulus load bearing primary strips are arrested when they run into the buffer strips. Using this technique, structures can be built in which cracks would be arrested before entire structural components failed. It appears that this could be an effective and efficient way to increase battle damage tolerance and the capability of a structural component to function after crack initiation.

#### D. RANGE OF THE STUDY

NARMC0 5208/T300  $[0/\pm 45]$  buffer strip and non-buffer strip joints were designed for widely varying laminate compositions, numbers and sizes of bolts. To simplify joint fabrication, the joint thickness was varied linearly between the inboard and outboard thicknesses, and all bolt holes for any given joint design were of the same diameter. To interface with the aluminum alloy fittings, the interbolt strain level was held to a maximum of 3000 micro-inches per inch, and the interbolt spacing was set at four hole diameters. The designs were compared to determine the effects of variation of hole size, number of holes, and laminate composition upon joint weight and excess bearing capacity.

Joint weight was considered a measure of the joint structural efficiency. Joint excess bearing capacity was considered a measure of allowable fabrication error. Since drilling holes in fibrous laminated composites has been



found to be both difficult and expensive, it was felt that the number of holes in any joint would be a measure of the relative joint fabrication cost (Ref. 5).

## II. ANALYSIS OF THE NON-BUFFER STRIP JOINT

### A. SIZING THE NON-BUFFER STRIP JOINT

Figure 1 is a schematic drawing showing the wing skin configuration used in the non-buffer strip joint design. All bolt holes are of the same diameter. For this analysis, wing taper is disregarded and the bolt hole centers are assumed placed in parallel rows and columns four hole diameters apart.

In the following theoretical development, it is assumed that the applied tensile load,  $N_x$ , and the applied shear load,  $N_{xy}$ , are constant across the outboard edge of the joint. Thus it is possible to size the joint considering only one column of bolts. It is further assumed that fittings were designed so that each bolt transfers the same portion,  $P$ , of the applied tensile load and that each bolt in the inboard row transfers, in addition to  $P$ , the same portion  $P_s$  of the shear load. In actual practice it is doubtful that this idealization could be achieved. However, it is a standard design assumption used in industry today (Ref. 8). This assumption implies a high resultant bolt load,  $R$ , in the inboard row of bolts.

From experimental analysis, it was known that layups of all  $\pm 45$  degree laminae would have a superior bolt load

bearing capacity (Ref. 9). Since the wing skin would be required to carry no applied load beyond the inboard row of bolts, it was decided to take advantage of this high bearing load capacity for all designs by requiring that, at the inboard row of bolts, the skin be composed of 100 per cent  $\pm 45$  degree plies.

Considering a four-hole-diameters-wide column of bolts, the total tensile load  $L$  in the skin is given by

$$(1) \quad L = N_x 4D$$

where  $D$  is the bolt diameter used in the joint. If there are  $N$  bolts in each column,

$$(2) \quad P = \frac{L}{N} = \frac{N_x 4D}{N}$$

As explained in Ref. 10, the bearing stresses in a plate due to bolt loads are a function of the magnitude of the bolt load, the diameter of the bolt hole, and the effective thickness of the plate. For this study  $t^*$ , the effective bolt bearing thickness of the plate, is defined as follows:

$$\text{for } t \leq 2D, \quad t^* = t$$

$$\text{for } t > 2D, \quad t^* = 2D$$

where  $t$  is the plate thickness. This definition of effective bearing plate thickness is adopted to account for the fact that in thick plates loaded through a bolt hole the bolt loads tend to distribute themselves so that higher portions of the load are carried at the plate edges than at the plate center. The thickness definition was not chosen

through rigorous experimental or analytical processes but rather in light of engineering experience with metal plates. The bolt bearing stress,  $\sigma_{BR}$ , is

$$(3) \quad \sigma_{BR} = \frac{\text{Bolt Load}}{D \ t^*}$$

On all but the inboard row of bolts,

$$(4) \quad \sigma_{bx} = \frac{4 \ N_x}{N \ t^*}$$

Since the inboard bolts in the non-buffer strip joints are assumed to react the shear as well as a share of the tensile load, they carry a bolt load, R, given by:

$$(5) \quad R^2 = \left( \frac{N_x \ 4D}{N} \right)^2 + (N_{xy} \ 4D)^2$$

$$(6) \quad R = 4DN_x \left[ \left( \frac{1}{N} \right)^2 + \left( \frac{N_{xy}}{N_x} \right)^2 \right]^{\frac{1}{2}}$$

The bearing stress on these inboard bolts is

$$(7) \quad \sigma_{BR} = \frac{4N_x}{t^*} \left[ \left( \frac{1}{N} \right)^2 + \left( \frac{N_{xy}}{N_x} \right)^2 \right]^{\frac{1}{2}}$$

Test specimens composed of NARMCO 5208/T300  $[\pm 45]$  layers were found to be able to withstand

$$\sigma_{BR}^{\max} = 78000 \text{ lbf./in.}^2$$

when this load was applied through untorqued bolts (Ref. 9). This value is used in Eq. 7 to determine the minimum allowable joint thickness at the inboard bolt holes. In cases where this is a critical design parameter, the geometry illustrated in Fig. 2(a) is used to size the joint. Otherwise, the geometry illustrated in Fig. 2(b) is used.

The bolt hole center locations are numbered from one to N beginning with location 1 at the first or outboard bolt and ending with location N at the last or inboard bolt. To simplify the analysis, other locations in the joints are specified by a parameter  $\eta$ , defined relative to the bolt hole numbers.  $\eta = 1$  means at the position of the first bolt hole center,  $\eta = 2$  means at the position of the second bolt hole center, and  $\eta = N$  means at the position of the inboard bolt hole center. Intermediate positions are defined as follows:

$\eta = 1.5$  means halfway between  $\eta = 1$  and  $\eta = 2$

$\eta = 2.5$  means halfway between  $\eta = 2$  and  $\eta = 3$

$\eta = N-.5$  means halfway between  $\eta = N-1$  and  $\eta = N$

The bypass force,  $F_{BP}$ , in the joint is defined as the tensile load passed from station  $\eta$  to station  $\eta+1$ , thus,

$$F_{BP} = N_x 4D \left(1 - \frac{\eta}{N}\right) \quad (\eta = \text{integer})$$

The bypass stress,  $\sigma_{tx}$ , is defined as

$$\sigma_{tx} = \frac{F_{BP}}{4Dt}$$

The bypass strain  $\epsilon$  is defined as

$$\epsilon = \frac{\sigma_{tx}}{E_x}$$

where  $E_x$  is the tensile modulus of the material.

The strain level halfway between station  $\eta$  and  $\eta+1$  is given by

$$(8) \quad \epsilon_{\eta+.5} = \frac{N_x (1 - \frac{\eta}{N})}{t E_x \eta+.5} \quad (\eta = \text{integer})$$

Both thickness and tensile modulus of the material are allowed to change with location within the joint.

The stress-strain behavior of a zero degree NARMCO 5208/T300 lamina is linear to failure. As reported in Ref. 2, the modulus of elasticity of such a lamina in the direction of the graphite fibers was determined by experiment to be  $20.5 \times 10^6$  lbf./in.<sup>2</sup>. The stress-strain behavior of a NARMCO 5208/T300 45 degree lamina is not linear. Tests performed in 1974, (Ref. 11), for a NARMCO 5208/T300 45 degree lamina under room temperature dry conditions reported the stress-strain behavior in the form of a secant modulus, which varied with strain level from  $2.9 \times 10^6$  lbf./in.<sup>2</sup> at zero strain to  $1.3 \times 10^6$  lbf./in.<sup>2</sup> at a strain level of 13000 micro-in./in. as shown in Fig. 3. (Secant modulus is the slope of a line through a point on the stress-strain curve and the origin.)

As explained in Ref. 3, the strain state for a balanced laminate composed of n layers is described by

$$(9) \quad \begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{14} \\ A_{12} & A_{22} & A_{24} \\ A_{14} & A_{24} & A_{44} \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_{xy} \end{bmatrix}$$

where  $[N]$  is a vector of applied tensile and shear loads and  $[\epsilon]$  is a vector describing the strain state of the plate. The components of the  $[A]$  matrix are defined as

follows:

$$(10) A_{ij} = \sum_{k=1}^n \bar{C}_{ij}^k t_k$$

where  $\bar{C}_{ij}^k$  are the elements of the compliance matrix of the  $k^{\text{th}}$  lamina.

This study is concerned only with balanced design laminates made from laminae of uniform thickness and material composition and oriented at either zero or  $\pm 45$  degrees to the spanwise direction. For such laminates, it is seen from Eqs. 9 and 10 that the various moduli vary linearly with the per cent of zero degree plies in the laminate. Because the  $\pm 45$  degree data on secant modulus was available, and because the secant modulus is convenient for design use, a laminate value of secant modulus was calculated. Figure 4 shows the variation of secant modulus with laminate composition for NARMCO 5208/T300  $[0/\pm 45]$  material at  $\epsilon = 3000$  micro-in./in. It was prepared assuming that this modulus varies linearly between the experimentally determined values for such laminates with zero and 100 per cent zero degree plies. Figure 4 is used to determine the tension modulus of the various NARMCO 5208/T300  $[0/\pm 45]$  laminates used in the study joint designs.

The laminate composition at station  $\eta = 1.5$  is assumed to be the same as that of the wing skin outboard of the joint. It determines the tensile modulus at this position. Equation 8 is then used to determine  $t_{1.5}$ .

The laminate composition at station  $\eta = N-.5$  is initially

determined by assuming that the percentage of zero degree plies varies linearly from station  $\eta = 1.5$  to station  $\eta = N$  where the laminate is to be composed of 100 per cent  $\pm 45$  degree plies. An additional constraint, applicable only to the non-buffer strip joint, is that the laminate at station  $\eta = N-.5$  can have no fewer than 5 per cent zero degree plies. This is done to ensure that there are sufficient load bearing zero degree plies to carry the bypass load between the second to last and the last, or inboard, row of bolts.

Having fixed the laminate composition, the tensile modulus and, through Eq. 8, the thickness at station  $N-.5$  are determined.

The remaining joint thicknesses are determined assuming a linear variation of joint thickness between  $\eta = 1.5$  and  $\eta = N-.5$ . Laminate compositions midway between each pair of adjacent bolts are determined using the thickness distribution and the desired strain level.

$$(11) \quad E_{x\eta+.5} = \frac{N_x (1 - \frac{\eta}{N})}{t_{\eta+.5} \epsilon} \quad (\eta = \text{integer})$$

The tension modulus,  $E_x$ , of the material is assumed to vary linearly with distance between the values determined from Eq. 11. Thus,

$$(12) \quad E_{x\eta} = \frac{E_{x\eta-.5} + E_{x\eta+.5}}{2} \quad (\eta = \text{integer})$$

These modulus values determine the laminate composition throughout the joint.



Joints are sized using 0.25, 0.375, 0.4375, and 0.5 inch bolts with laminate compositions at the first bolt varying from 60 to 10 per cent zero degree plies. These laminate compositions cover the range over which the Eisenmann strength model to be described in the next section was considered accurate. In all cases, the same interbolt strain level, 3000 micro-inches per inch, is maintained. The minimum number of bolts used in any joint design is three. This is done to provide a mechanism by which the desired strain level can be maintained. The maximum number of bolts in any joint is determined by the desired interbolt spacing of four bolt hole diameters and the requirement that the joint length not exceed ten inches.

#### B. DETERMINING THE STRENGTH OF A NON-BUFFER STRIP JOINT

Waddoups, Eisenmann, and Kaminski, in Ref. 12, showed experimentally that graphite-epoxy laminates are statically brittle and exhibit many of the failure characteristics of brittle materials first explained by Griffith in Ref. 13. They formulated a model which assumed that crack growth behavior in graphite-epoxy laminates was a function of stress intensity and critical energy level, and they verified their model by experiment.

In Ref. 14, Eisenmann continued this work and developed a bolted joint strength model for composite materials. This model accounted for the material ultimate strengths, local stress intensity factors, and geometric width correction factors. With the Eisenmann model, it was



possible to calculate the total stress at any point on a loaded circular hole in an orthotropic plate by linearly combining the various stresses acting upon the plate using Eq. 13. Because of the internal curve fitting techniques used in this model, it was considered to give accurate results for laminate compositions varying from 10 to 60 per cent zero degree plies (Ref. 14).

$$\begin{aligned}
 (13) \quad \sigma^i = & \lambda_{tx}^i f\left(\frac{a}{r}\right)_i^i \sigma_{tx} + \lambda_{ty}^i f\left(\frac{a}{r}\right)_i^i \sigma_{ty} \\
 & + \lambda_{xy}^i f\left(\frac{a}{r}\right)_i^i \sigma_{xy} + \lambda_{bx}^i f\left(\frac{a}{r}\right)_i^i \sigma_{bx} \\
 & + \lambda_{by}^i f\left(\frac{a}{r}\right)_i^i \sigma_{by}
 \end{aligned}$$

where:

$i$  = indicator of the exact position on the hole.

$\lambda_{\alpha}^i$  = finite width correction factor at location  $i$  for applied load condition  $\alpha$

$f\left(\frac{a}{r}\right)_i^i$  = effective isotropic stress concentration factor at location  $i$  for applied load condition  $\alpha$

$\sigma_{\alpha}$  = applied stress for load condition  $\alpha$

$\sigma^i$  = net tangential stress at location  $i$

$\alpha$  = subscript denoting applied load condition as follows:

$xy$  = shear

$tx$  = tension in  $x$  - direction

$ty$  = tension in  $y$  - direction

$bx$  = bearing in  $x$  - direction

by = bearing in y - direction

The stress definitions are sketched in Fig. 5. The  $\lambda$  and  $f$  factors are determined by plate geometry and material composition.

The failure modes of NARMCO 5208/T300  $[0/\pm 45]$  plates with loaded holes were determined by test (Ref. 14). Crack initiation in the test specimens most often occurred on the hole edge at positions  $\beta=0, \pm 45, \pm 90, \pm 135$ , or  $180$  degrees measured from the X axis. From symmetry it was determined that all these failure modes could be adequately described by description of the failure modes encountered at  $\beta = 0, 45$ , and  $90$  degrees. Laminate strength for each of these positions was determined by experiment (Ref. 14). In these tests the bolts used to load the holes were untorqued.

The Eisenmann static strength model was used to calculate stress intensity and geometric width correction factors based upon an interbolt spacing of four hole diameters. Equation 13 was used to prepare Figs. 6-29 which define the failure modes expected for laminates whose composition varies from 60 to 10 per cent zero degree plies with 0.25, 0.375, 0.4375, and 0.50 inch holes. Only the effects of  $\sigma_{xy}$ ,  $\sigma_{tx}$ , and  $\sigma_{bx}$  were considered in the preparation of these curves.

One of the parameters of interest in this study is the excess bearing capacity of each joint design. For purposes of this study the excess bearing capacity of the joint is defined as the smallest excess bearing capacity at any

bolt hole in the joint. At each hole a representative stress load,  $S_1$ , which accounts for the combined bearing and bypass stresses is calculated as follows:

$$(14) \quad S_1 = \left[ \sigma_{bx}^2 + \sigma_{tx}^2 \right]^{\frac{1}{2}} \quad \text{as loaded}$$

A representative ultimate strength,  $S_2$ , is calculated from:

$$(15) \quad S_2 = \left[ \sigma_{bx}^2 + \sigma_{tx}^2 \right]^{\frac{1}{2}} \quad \text{at failure}$$

where the failure state is the state at which

$$(16) \quad \left[ \frac{\sigma_{bx}}{\sigma_{tx}} \right]_{\text{failure}} = \left[ \frac{\sigma_{bx}}{\sigma_{tx}} \right]_{\text{loaded}}$$

Then, as shown in Fig. 30, the excess bearing capacity,  $B$  is defined as

$$(17) \quad B = \frac{S_2 - S_1}{S_1}$$

$B > 0$  at all holes in a joint implies that  
there is some margin of safety.

$B = 0$  at any hole in a joint implies that  
there is no margin of safety.

$B < 0$  at any hole in a joint implies that  
the joint would fail under the applied  
load.

The effect of laminate composition and bolt hole size on excess bearing capacity is shown in Figs. 31, 32, and 33.

#### C. DETERMINING THE WEIGHT OF A NON-BUFFER STRIP JOINT

The weights per inch of chord for the non-buffer strip joints were calculated by multiplying the cross-sectional

areas of each joint by the density of NARMCO 5208/T300 graphite-epoxy laminate material. The effect of laminate composition, hole size, and number of bolt holes upon joint weight is shown in Figs. 34, 35, and 36.

### III. ANALYSIS OF THE BUFFER STRIP JOINT

#### A. SIZING THE BUFFER STRIP JOINT

Figure 37 is a schematic of the buffer strip joint design. All bolt holes are of the same diameter. Except for the inboard row, all bolt holes are placed in the buffer strips. This placement was chosen for two reasons:

1. It took advantage of the high bearing capacity of NARMCO 5208/T300  $[\pm 45]$  laminates.
2. It reduced stress concentrations in the heavily loaded primary strips.

The bolt holes in the buffer strips are spaced so that there is a distance of four hole diameters between adjacent hole centers. Two bolt holes are placed in each primary strip which is located between two buffer strips. These bolt holes are placed in a row with the inboard bolt in the buffer strip. In the joint analysis, wing taper is disregarded and the spanwise edges of the buffer and primary strips are considered parallel. It is assumed that joint thicknesses at any position are the same in the buffer and primary strips.

The buffer strips used in the joints analyzed in this study are four hole diameters wide, and the two primary

strips are each 3.335 hole diameters wide. This gives an overall buffer strip joint width of 10.67 hole diameters. These dimensions were chosen because it was felt that they were representative of a geometry which could be used in an advanced fighter type aircraft wing skin application. No attempt is made to justify these dimensions either analytically or experimentally, and no attempt is made to assess the effect of this choice of dimensions upon joint efficiency.

In the following theoretical development, as in the case of the non-buffer strip joint, a tensile load  $N_x$  and a shear load  $N_{xy}$  are assumed constant across the outboard edge of the joint. It is also assumed that fittings were designed so that each row of bolts in the joint transfers the same portion  $P$  of the applied tensile load and so that the shear load is reacted by the inboard row of bolts, each of the inboard bolts carrying an equal share of the shear load as well as a share of the tensile load. To utilize the high bearing capacity of a NARMCO 5208/T300  $[\pm 45]$  laminate, it was decided to impose a requirement that at the inboard row of bolts, the primary strips be composed of 100 per cent  $\pm 45$  degree plies.

Considering the joint sketched in Fig. 37, the load,  $L$ , on a single buffer strip joint, is given by:

$$(18) \quad L = N_x AD$$

where  $AD$  is the overall joint width. The bolt load in all bolts except those in the inboard row is given by

$$(19) \quad P = \frac{L}{M} = \frac{N_x AD}{M}$$

where M is the number of rows of bolts in the joint.

Since the shear load  $N_{xy}$  and a total tensile bolt load P are reacted equally by each of the three bolts in the inboard row of bolts, the resultant load, R, on each of these bolts is calculated from

$$(20) \quad R^2 = \left[ \frac{N_x AD}{3(M)} \right]^2 + \left[ \frac{N_{xy} AD}{3} \right]^2$$

$$(21) \quad R = \frac{ADN_x}{3} \left[ \left[ \frac{1}{M} \right]^2 + \left[ \frac{N_{xy}}{N_x} \right]^2 \right]^{\frac{1}{2}}$$

As in the non-buffer strip joint design, the desired interbolt strain level, 3000 micro-inches per inch, is the primary consideration determining the joint geometry for the buffer strip design. Locations in the joint are described by station numbers, just as in the non-buffer strip design. In the case of the buffer strip design, however, the station numbers vary from  $\eta=1$ , which corresponds to the location of the outboard bolt-hole center, to station  $\eta=M$  which corresponds to the location of the row of centers of the inboard bolts.

The thickness at station  $\eta=1.5$  is determined by the applied load, the joint geometry, the joint material composition, and the desired interbolt strain level. The average modulus of the joint,  $E_{x_{ave}}$ , as defined in Ref. 7, is used in calculating this thickness.

$$(22) \quad E_{x_{ave}} = \frac{D(A-W_B) E_{x_1} + W_B D E_{x_2}}{AD}$$

where  $W_B D$  = width of the buffer strip

$E_{x_1}$  = tensile modulus of the primary strip

$E_{x_2}$  = tensile modulus of the buffer strip

As shown in Fig. 4,  $E_{x_1}$  is determined by the percentage of zero degree plies in the primary laminate. As shown in Fig. 3,  $E_{x_2}$  varies with strain level.

The thickness at station  $\eta=1.5$  is derived from

$$(23) \quad \epsilon = \frac{AD N_x \left[1 - \frac{1}{M}\right]}{AD t E_{x_{ave}}}$$

where  $\epsilon$  is the desired interbolt strain level. Thus,

$$(24) \quad t_{1.5} = \frac{AN_x \left[1 - \frac{1}{M}\right]}{\left[(A-W_B)E_{x_1} + W_B E_{x_2}\right] \epsilon}$$

Similarly,

$$(25) \quad t_{M-.5} = \frac{AN_x \left[\frac{1}{M}\right]}{\left[(A-W_B)E_{x_1} + W_B E_{x_2}\right] \epsilon}$$

The bolt bearing stress in the inboard row of bolts is given by

$$(26) \quad \sigma_{BR} = \frac{R}{t^*}$$

where  $t^*$ , the effective bearing thickness, is defined as in the non-buffer strip joint.

As stated earlier, the bearing strength of NARMCO 5208/T300  $[\pm 45]$  degree laminates is 73000 lbf./in.<sup>2</sup>. This determines the minimum allowable joint thickness at station



$\eta = M$  for any given load condition.

The laminate composition in the primary strips is initially assumed to vary linearly with distance from the composition at station  $\eta = 1.5$ , where it is the same as that of the plate outboard of the joint, to 100 per cent  $\pm 45$  degree plies at station  $\eta = M$  where the applied shear loads are reacted.

The joint thickness at station  $\eta = M-.5$  is determined by the desired interbolt strain level.

$$(27) \quad t_{M-.5} = \frac{AN_x}{M[(A-W_B) E_{x_1} + W_B E_{x_2}] \epsilon}$$

The remaining joint thicknesses are then determined geometrically from those at  $\eta = 1.5$  and  $\eta = M-.5$  using the same techniques as for the non-buffer strip joint. If possible, a cross-section similar to that shown in Fig. 2(b) is used. When this yields a thickness at the last bolt which is too small, using the maximum bearing stress criteria discussed above and Eq. 26, a cross-section similar to that of Fig. 2(a) is used.

Having fixed the joint geometry, the laminate composition is determined by the requirement that the design strain level be maintained between each pair of bolt holes.

At any station  $\eta = k+.5$ ,  $k=1, 2, \dots, M-1$ ,

$$(28) \quad \epsilon = \frac{N_x A [1 - \frac{k}{M}]}{[E_{x_1} (A - W_B) + E_{x_2} W_B] t}$$

Since the composition of the buffer strips is fixed and  $E_{x_2}$



is determined by the interbolt strain level, it is necessary to vary  $E_{x_1}$  and hence the composition of the primary strips to maintain the desired strain level as the bypass loads in the joint vary from hole to hole.

$$(29) \quad E_{x_1} = \frac{1}{(A-W_B)} \left[ \frac{N_x A \left[ 1 - \frac{k}{M} \right]}{t \eta \epsilon} - W_B E_{x_2} \right]$$

It is assumed that  $E_{x_1}$  varies linearly with distance between stations  $\eta = k-.5$  and  $\eta = k+.5$ . Thus

$$(30) \quad E_{x_1 k} = \frac{E_{x_1 k+.5} + E_{x_1 k-.5}}{2}$$

Determination of  $E_{x_1 k}$  determines the required percentage of zero degree plies in the primary laminate at station k.

Having determined the joint geometry and laminate composition, it is then possible to determine the joint weight and excess bearing capacity.

## B. DETERMINING THE STRENGTH OF A BUFFER STRIP JOINT

The bolt load  $P$  is not carried entirely in the buffer strip. Part of it is transmitted, in shear, to the primary strips. This load splitting is shown in Fig. 38 in which  $P_1$  is the portion of the bolt load  $P$  reacted through each of the primary strips and  $P_2$  is the portion of  $P$  reacted through the buffer strip. The relationship between  $P_1$  and  $P_2$  is determined analytically as follows:

Referring to Fig. 38,

$$(31) \quad P = 2P_1 + P_2$$

The bypass strain levels in the buffer and primary strips

are assumed the same. Defining

$$(32) \quad 2W_1D = AD - W_BD$$

$$(33) \quad 2W_1 = (A - W_B)$$

$$(34) \quad \frac{P_1}{E_{x_1}W_1t} = \frac{P}{2E_{x_1}W_1t + E_{x_2}W_Bt}$$

Rearranging Eq. 34 gives an expression for  $P_1$  in terms of  $P$ , the joint composition, and the joint geometry.

$$(35) \quad \frac{P_1}{P} = \frac{E_{x_1}W_1t}{2E_{x_1}W_1t + E_{x_2}W_Bt}$$

This equation is rewritten in the form

$$(36) \quad P_1 = A_n P$$

where  $A_n$  is determined by knowledge of the joint laminate composition, geometry, and strain level.

$$(37) \quad A_n = \frac{E_{x_1}W_1}{2E_{x_1}W_1 + E_{x_2}W_B}$$

The bolt load splitting discussed above is dependent upon the ability of the  $\pm 45$  degree laminae to transfer a shear load,  $F_S$ , from the buffer to the primary strips. It was experimentally determined that failure of this load transferring mechanism could be expected when the shear stress in these fibers,  $f_S$ , reached a magnitude of 90,000 lbf./in.<sup>2</sup> (Ref. 9). At any station in the joint

$$(38) \quad F_{S_\eta} = \frac{P_1}{4Dt_{\pm 45}} \quad (\eta = \text{integer})$$

where  $t_{\pm 45}$  is the total thickness of  $\pm 45$  degree laminae through which  $P_1$  is transferred. Defining  $Z$  = percentage of zero degree plies in the primary strips,

$$(39) \quad t_{\pm 45} = t \frac{(100-Z)}{100}$$

$$(40) \quad f_{s\eta} = \frac{N_x A A_n}{4M t \frac{(100-Z)}{\eta} \frac{100}{100}} \quad (\eta = \text{integer})$$

The shear stress  $f_s$  varies from bolt to bolt in a given buffer strip joint design. In all designs, however, the highest values for  $f_s$  occur at the first bolt hole. Thus  $f_{s1}$  determines the upper limit on the percentage of zero degree plies in the primary strip laminate. From Eq. 24,

$$(41) \quad t_1 = \frac{AN_x \left[ 1 - \frac{1}{M} \right]}{\left[ (A-W_B)E_{x1} + W_B E_{x2} \right] \epsilon}$$

Fig. 4 yields the following relationship between laminate composition and modulus,

$$(42) \quad E_{x1} = 10^6 \left[ 20.7 - (100-Z) \left[ \frac{20.7-2.3}{100} \right] \right] (\text{lb./in.}^2)$$

Then,

$$(43) \quad \frac{100-Z}{100} = \left[ 20.7 - \frac{E_{x1}}{10^6} \right] \frac{1}{17.9}$$

Substituting Eqs. 37, 41, and 43 into Eq. 40,

$$(44) \quad f_{s1} = \frac{W_1 E_{x1}}{(M-1)} \frac{1}{4} \frac{\epsilon_1}{\left[ 20.7 - \frac{E_{x1}}{10^6} \right]} \frac{1}{17.9}$$

From Eq. 44 it is seen that  $f_{s1}$  is determined by the joint

geometry, laminate composition, and design strain level. Fixing the joint geometry and the design strain level determines the maximum allowable modulus for the primary strips and hence provides an upper limit on the per cent zero degree plies which can be used in the primary strips. This limit is determined by setting  $f_{s_1}$  equal to its experimentally determined maximum, 90,000 lbf./in.<sup>2</sup> and using Eq. 45 to determine  $E_{x_{1_{\max}}}$ .

(45)

$$E_{x_{1_{\max}}} = \frac{4(M-1)(20.7)f_{s_1}}{17.9(W_1)(\epsilon_1)10^6 + 4(M-1)f_{s_1}} 10^6 (\text{lbf./in.}^2)$$

Under the design conditions applicable to this study, Eq. 45 implies an upper limit of 92 to 98 per cent zero degree plies in the primary strips.

Three other failure modes of a buffer strip joint were found to be most probable under  $N_x$  and  $N_{xy}$  loading (Ref. 15). Type I and Type II failure occurred in the buffer strip at the loaded bolt holes. Type I failure was characterized by radial cracks at 45 degrees to the X axis. Type II failure was characterized by radial cracks originating at the edge of the hole at 90 degrees to the X axis. Type III failure occurred when the load bearing fibers in the primary strips were broken. These three failure modes are sketched in Fig. 39.

The Type I failure mode characterizes the interaction of bearing and shear stresses in the buffer strip. It was found to be essentially independent of the bypass

stress. The load curves describing this failure mode were derived from experimental results (Ref. 15). Single hole specimens of buffer strip joint material were clamped in test machines along either one or two sides as shown in Fig. 40. With the bolts torqued, the specimens were loaded, and the failure bolt load stresses measured. Tests were run for specimens with 0.25 inch and 0.4375 inch diameter holes. For the doubly clamped test cases Type I failure occurred under the following loads:

$$D = 0.250 \text{ in. } \sigma_{bx_{\max}} = 151,200 \text{ lbf./in.}^2$$

$$D = 0.4375 \text{ in. } \sigma_{bx_{\max}} = 144,100 \text{ lbf./in.}^2$$

For the test specimens clamped at only one edge, Type I failure occurred under the following loads:

$$D = 0.25 \text{ in. } \sigma_{bx} = 111,000 \text{ lbf./in.}^2$$

$$D = 0.4375 \text{ in. } \sigma_{bx} = 107,000 \text{ lbf./in.}^2$$

Satisfactory test results for the pure shear load case could not be obtained.

An attempt was made to approximate the shear effects by superposition of the singly and doubly clamped test results. The finite element computer program ISANIS, listed in Appendix A, was used to analyze the stress concentration field in various orthotropic plates. It was found that for a square plate made from uniform material with a central hole and sides at least four hole diameters in length, the stress field at the hole due to pure applied

shear could be closely approximated by an appropriate superposition of the stress fields resulting from singly and doubly clamped load cases. The superposition used is shown schematically in Fig. 41. The key assumption in this superposition is that the moment reaction which is representative of the single clamped edge load case can be replaced by a couple of equal magnitude resulting from shear loads applied on the upper and lower edges of the specimen. This assumption is really an application of the St. Venant principle that points in a body removed from load application locations react to the load applied rather than its mechanism of application. ISANIS was used to test this assumption and, under the conditions stated above, it was found to be reasonable.

The test specimens were thin and hence  $t^*=t$ . Therefore,

$$(46) \quad P = \sigma_{bx}Dt$$

For the double clamped specimen, the reaction force,  $F_{RD}$ , is given by

$$(47) \quad F_{RD} = \frac{P}{2} = \frac{\sigma_{bx}Dt}{2}$$

Assuming that this force is uniformly distributed across the clamped edges, an edge stress is defined

$$(48) \quad \sigma_{RD} = \frac{F_{RD}}{\ell Dt}$$

where  $\ell \times D$  equals the length of the clamped edge.

For the single clamped specimen, the reaction force,

$F_{RS}$ , is equal to the applied bolt load  $P$ . The reaction moment,  $m$ , is given by

$$(49) \quad m = P \frac{\ell D}{2}$$

This moment is approximated by a couple of the same magnitude formed by forces acting on the edges of the specimen which are normal to the clamped edge. For square specimens of side length  $\ell D$ , the magnitude of these forces,  $F$ , is given by

$$(50) \quad F \ell D = m$$

$$(51) \quad F \ell D = \frac{P \ell D}{2}$$

$$(52) \quad F = \frac{P}{2}$$

This development is also shown schematically in Fig. 41.

Assuming that each force  $F$  is distributed uniformly along the specimen edge upon which it is applied, an edge shear is defined

$$(53) \quad \sigma_{xy} = \frac{F}{\ell D t}$$

$$(54) \quad \sigma_{xy} = \frac{P}{2 \ell D t}$$

For the test specimens each side was four hole diameters in length. From Eqs. 47 and 54, the superposition yields

$$(55) \quad \sigma_{xy} = \frac{\sigma_{bx}}{8}$$

Under these assumptions, the experimental data listed previously yield the following failure states:



$$D = 0.25 \text{ in.}$$

$$\begin{array}{ll} \sigma_{xy} = 0 & \sigma_{bx} = 151,200 \text{ lbf./in.}^2 \\ \sigma_{xy} = 13,875 \text{ lbf./in.}^2 & \sigma_{bx} = 111,000 \text{ lbf./in.}^2 \end{array}$$

$$D = 0.4375 \text{ in.}$$

$$\begin{array}{ll} \sigma_{xy} = 0 & \sigma_{bx} = 144,100 \text{ lbf./in.}^2 \\ \sigma_{xy} = 13,375 \text{ lbf./in.}^2 & \sigma_{bx} = 107,000 \text{ lbf./in.}^2 \end{array}$$

Figure 42 was prepared from these data points assuming that the ultimate bearing stress-shearing stress interaction curve was linear for the graphite-epoxy laminates used in this study.

Buffer strip Type II failure curves were experimentally determined for buffer strip joint specimens with 0.4375 inch bolt holes, a buffer strip width of 1.5 inches, and primary strips each 1.25 inches wide (Ref. 15). The hole centers were spaced four diameters apart. These tests were run with primary strips composed of 30 per cent zero degree plies and 50 per cent zero degree plies with bolt loads applied through torqued bolts. These test specimens were thin enough so that  $t^*=t$ . Tests were also run on buffer strips alone to determine the ultimate bypass stress of the buffer strip material (Ref. 15). With no applied bearing stress it was found that the ultimate bypass stress in the buffer strip material was 25000 lbf./in.<sup>2</sup>. The results of this series of tests indicated that the Type II failure mode for these test specimens could be closely described by the empirical relationship



$$(56) \quad \sigma_{\text{net buffer}} = \frac{P_2 + .25(2P_1)}{D(W_B - 1) t_2} + \sigma_{\text{tx}_2}_{\text{net area}}$$

$\sigma_{\text{net buffer}}$  is the ultimate bypass stress in the buffer strip normalized to the width of the buffer strip minus the diameter of the bolt hole.  $\sigma_{\text{tx}_2}_{\text{net area}}$  is the actual bypass stress in the buffer strip normalized to the width of the buffer strip minus the hole diameter. Thus,

$$(57) \quad \sigma_{\text{net buffer}_{\text{max}}} = 25000 \text{ lbf./in.}^2 \left( \frac{W_B}{W_B - 1} \right)$$

$$(58) \quad \sigma_{\text{tx}_2}_{\text{net area}} = \frac{\sigma_{\text{tx}_2} W_B}{(W_B - 1)}$$

Assuming that the strain levels in the buffer and primary strips are the same at any station  $\eta$ ,

$$(59) \quad \frac{\sigma_{\text{tx}_1}}{E_{x_1}} = \frac{\sigma_{\text{tx}_2}}{E_{x_2}} = \epsilon$$

Thus,

$$(60) \quad \sigma_{\text{tx}_2}_{\text{net area}} = \frac{\sigma_{\text{tx}_1}}{E_{x_1}} E_{x_2} \frac{W_B}{(W_B - 1)}$$

$$(61) \quad \sigma_{\text{bx}} = \frac{P}{Dt}$$

$$(62) \quad 25000 \left( \frac{W_B}{W_B - 1} \right) = \frac{\sigma_{\text{bx}} (1 - 1.5 A_n)}{(W_B - 1)} + \frac{\sigma_{\text{tx}_1} E_{x_2}}{E_{x_1}} \frac{W_B}{(W_B - 1)}$$

For a fixed laminate composition in the primary strips, different values of  $\sigma_{\text{tx}_1}$  produce different strain levels. This implies variation in  $E_2$ , the modulus of the buffer

strip material, with bypass stress in the primary strips. This variation in modulus explains why the Type II failure is not linear in  $\sigma_{bx}$  and  $\sigma_{tx_1}$ .

Type III failure is characterized by fracture of the zero degree fibers in the primary strips. As reported in Ref. 2, this failure mode is encountered when the strain level in these strips reaches 10,000 micro-inches per inch. This failure mode is mathematically predicted by considering the presence of both the bypass stress and the stress due to  $P_1$  in the primary strips. Thus,

$$(63) \quad \sigma_{tx_1} + \frac{A_n P}{W_1 D t} = \sigma_1$$

$$(64) \quad \sigma_{1_{ult}} = \epsilon_{ult} E_1$$

$$(65) \quad P = \sigma_{bx} D t^*$$

$$(66) \quad \sigma_{tx_1} + \frac{A_n \sigma_{bx} D t^*}{(A - W_B) D t} = \epsilon_{ult} E_1$$

The results of these tests are summarized in Fig. 43. The Type I failure line is drawn for a zero shear case and is determined from experiment with a double clamped test specimen, the Type II failure lines are drawn from Eq. 62, and the Type III failure lines are drawn from Eq. 66. For the joints used in this study, Eq. 62 becomes

$$(67) \quad \frac{\sigma_{bx}(1-1.5A_n)}{3} + \sigma_{tx_1} \frac{\frac{E_{x_2}}{E_{x_1}} \frac{4}{3}}{\frac{4}{3}} = 25 \frac{4}{3}$$

and Eq. 66 becomes

$$(63) \quad \sigma_{tx_1} + \frac{A_n \sigma_{bx}}{6.67} = \epsilon_{ult} E_1$$

Figures 44 and 45 describe the expected failure states for the buffer strips used in the study joints. The Type I failure lines on these figures are taken from the zero shear ultimate bearing stresses indicated on Fig. 42. The Type II and Type III failure lines shown in these figures are drawn from Eqs. 67 and 68.

Excess bearing capacity calculations were made for buffer strip joints just as had been done for non-buffer strip joints. In the case of the buffer strip joints, however, only 0.25 and 0.4375 inch holes were considered since Type I failure test data was available only for these hole sizes. The results of these calculations are presented in Figs. 46 and 47.

#### C. DETERMINING THE WEIGHT OF A BUFFER STRIP JOINT

The weight per inch of chord of each buffer strip design was calculated just as had been done for the non-buffer strip joints by multiplying the cross-sectional area of each joint by the density of the NARMCO 5208/T300 graphite-epoxy material used in the study. The variation of joint weight with primary strip laminate composition, bolt hole size, and number of bolts is shown in Figs. 48 and 49.

#### IV. DISCUSSION OF RESULTS

##### A. NON-BUFFER STRIP JOINTS

The following generalizations about non-buffer strip joints were found to be valid:

1. For a given hole size, the joint weight decreased as the percentage of zero-degree plies in the joint increased.
2. For a given hole size, the fewer the number of bolt holes per unit chord, the lighter the joint.
3. The smaller the bolt holes, the lighter the joint could be made.
4. The smaller the bolt holes, the greater the range in number of allowable bolt holes per unit chord.
5. For a given laminate composition, the smaller the bolt holes the larger the minimum number of bolt holes per unit chord required.
6. The smaller the bolt holes, the larger the allowable range of laminate composition.
7. For a given laminate composition, the smaller the bolt holes, the larger the excess bearing capacity.

##### B. BUFFER STRIP JOINTS

Observations 1-6, above, are also true for buffer strip joints. The excess bearing capacity of the buffer

strip joint, as seen in Figs. 48 and 49, is, in a gross sense, independent of hole size and more heavily influenced by laminate composition, the number of holes, and the load condition.

#### C. COMPARISON OF NON-BUFFER STRIP AND BUFFER STRIP JOINTS

In Ref. 16 it is explained that the bearing strength of bolted plates is higher when the bearing loads are applied through torqued bolts than when they are applied through untorqued bolts. Since the failure modes of the buffer strip joints were derived from experiments in which the bolt loads were applied through torqued bolts, and since the failure modes of the non-buffer strip joints were derived from experiments in which the bearing loads were applied through untorqued bolts, this may explain at least part of the apparently higher excess bearing capacities available with buffer strip joints. Buffer strip joints generally weigh more than non-buffer strip joints. The fact that buffer strip joints require fewer bolts per inch of chord than non-buffer strip joints can be used to offset some of this weight difference if bolt weights are included in the net joint weight. The reduced number of bolts per inch of chord possible with buffer strip joints should also reduce joint fabrication costs by reducing the number of drilling operations required. Buffer strip joints can be constructed for a larger range of laminate composition than non-buffer strip joints.

## V. CONCLUSIONS AND RECOMMENDATIONS

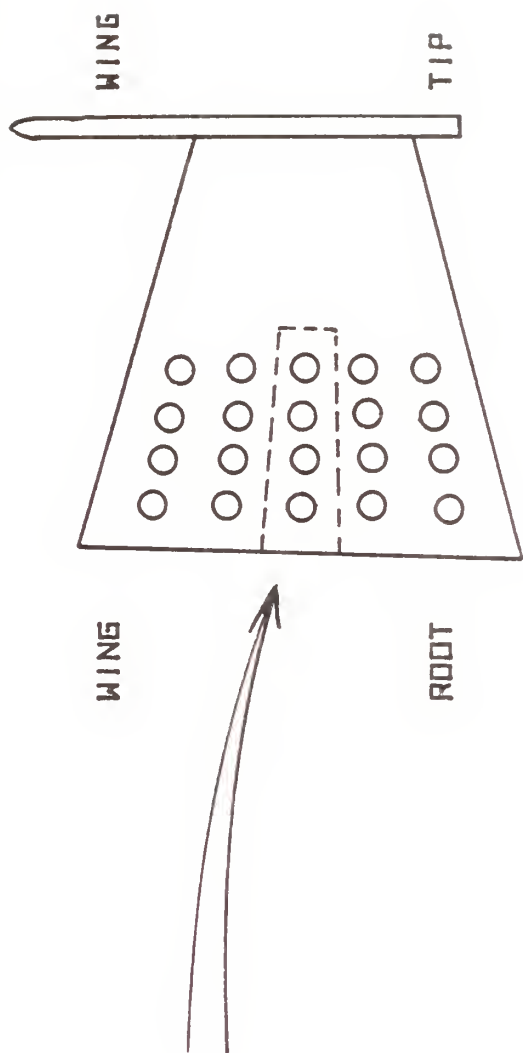
The design methodology used in this study is shown to be capable of producing workable joint designs. The wide range of weights and excess bearing capacities exhibited by the different joint designs indicates that the joint efficiencies achieved by geometric substitution of advanced composites for conventional structural materials were probably minimal when compared with those which would be achieved by designs which took advantage of the special high strength and high modulus properties of advanced composites.

Under the design conditions adopted in this study, buffer strip joints were found to be stronger and more cheaply manufactured than non-buffer strip joints. These advantages are offset by the increased joint weights characteristic of buffer strip joints. In spite of this weight penalty, it is felt that the high excess bearing capacity and integral crack stoppage capability of buffer strip joints makes them promising candidates for aerospace applications.

It is felt that Figs. 31-36 and 46-49 can be used to compare various design proposals and to estimate the costs of variation in laminate composition, hole size, or number of holes. No attempt was made to determine the effect of the design limitations summarized in TABLE I which were placed upon allowable joint geometry and

composition by such factors as manufacturing considerations and fitting interface requirements. It is recommended that the effect of these restrictions be determined by an analysis similar to this one with the restrictions removed.

It is recommended that application of the buffer strip technique to critical components be preceded by further experimentation to more accurately determine the behavior of buffer strip joints under shear loads.



PER CENT ZERO DEG. PLIES IN JOINT  
 DECREASES FROM VALUE AT OUTBOARD  
 BOLT TO ZERO AT INBOARD BOLT

FIGURE 1. SCHEMATIC OF A WING WITH A NON-BUFFER STRIP JOINT



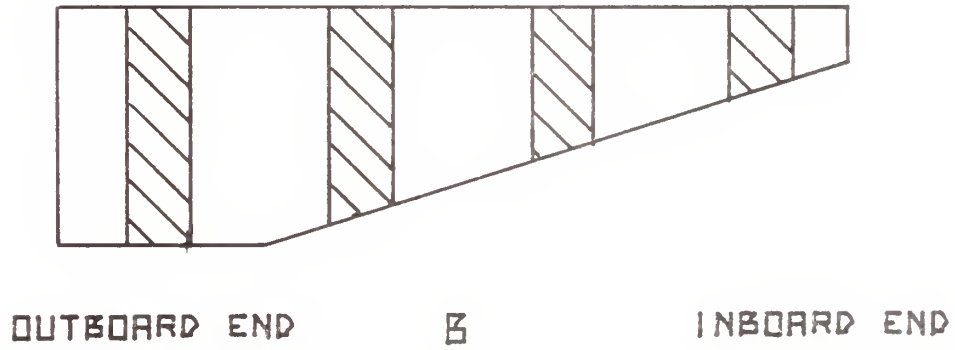
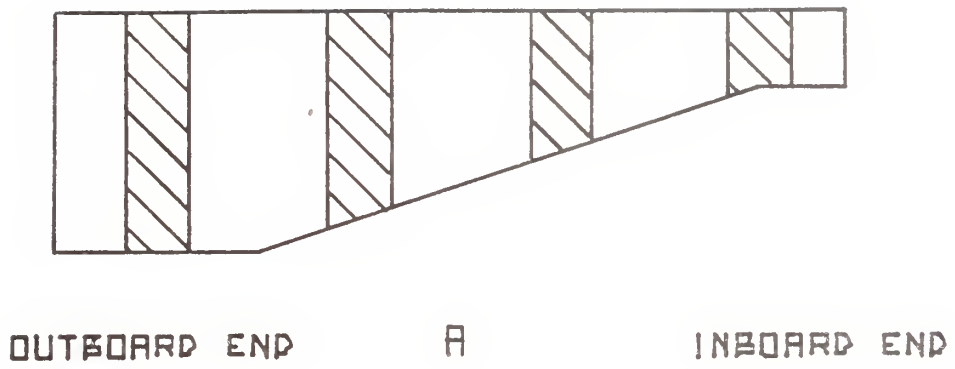


FIGURE 2. PERMISSIBLE JOINT CROSS SECTIONS

VARIATION OF SECANT MODULUS WITH STRAIN  
OF NARMCO 5208/T300 ( $\pm 45$  DEG.)  
LAMINATED MATERIAL AT ROOM  
TEMPERATURE

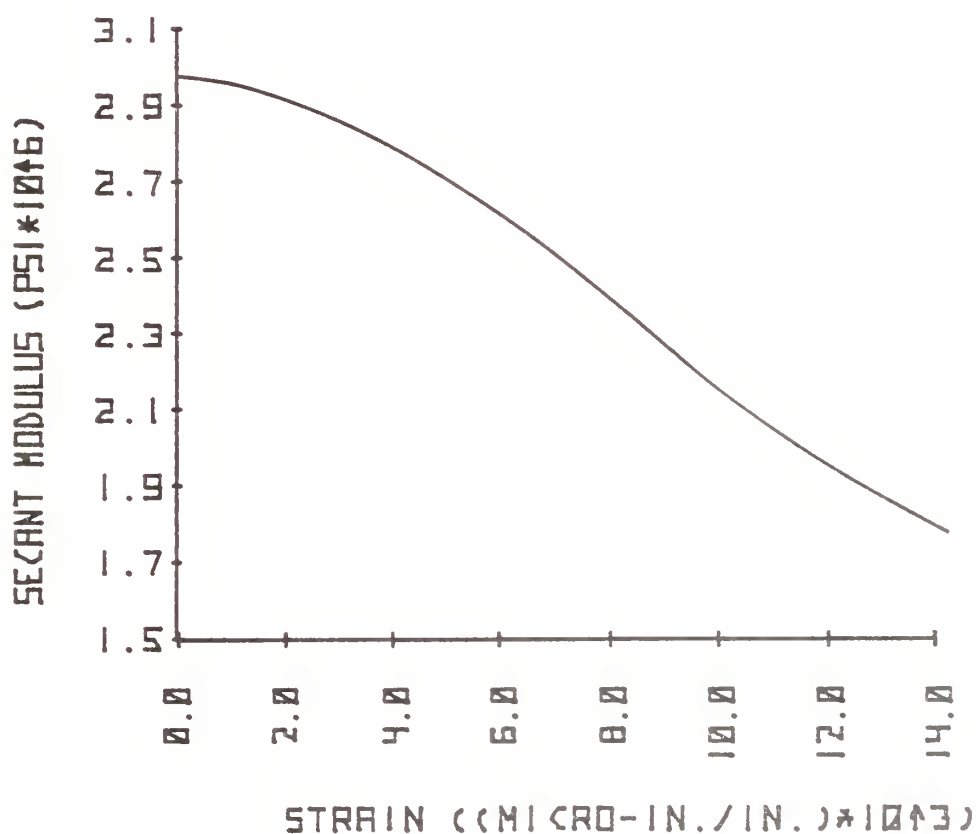


FIGURE 3. VARIATION OF SECANT MODULUS WITH STRAIN OF NARMCO  
5208/T300 ( $\pm 45$  DEG.) LAMINATED MATERIAL AT ROOM TEMPERATURE

# NARMCO 5208/T300 (0/±45) LAMINATES

E (SECANT MODULUS) VS. PERCENT  
ZERO DEGREE PLIES

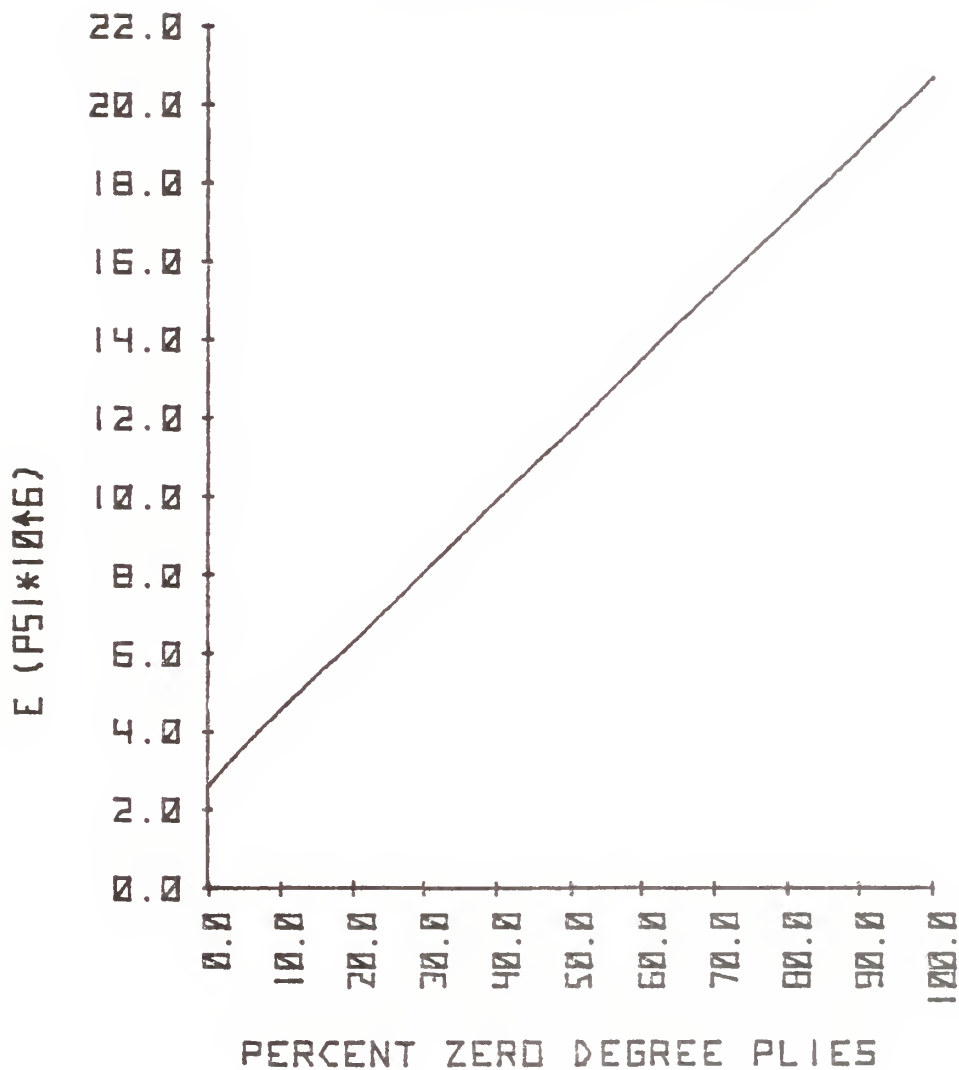


FIGURE 4. SECANT MODULUS OF NARMCO 5208/T300 [0/±45]  
LAMINATES VS. PER CENT ZERO DEGREE PLIES

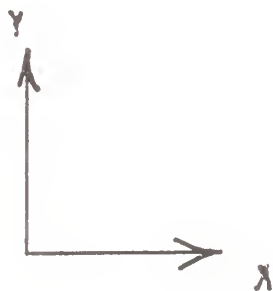
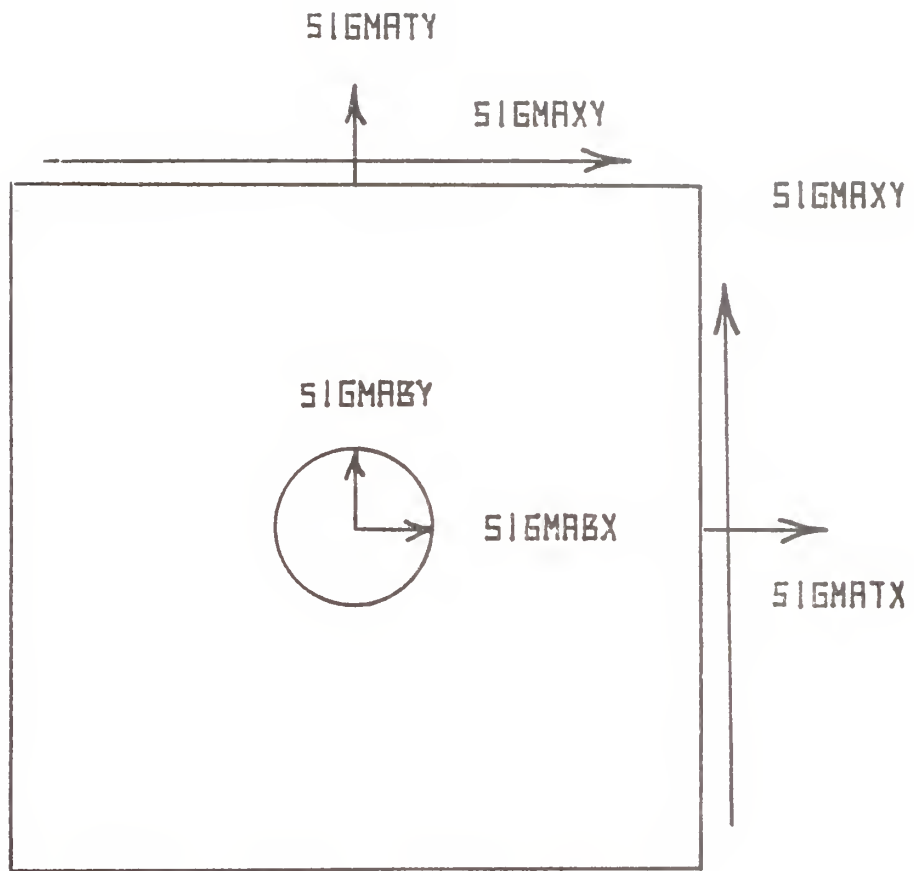


FIGURE 5. BOLTED JOINT APPLIED STRESS DEFINITIONS

0.25 IN. DIAM. HOLE  
10% ZERO DEG. PLIES

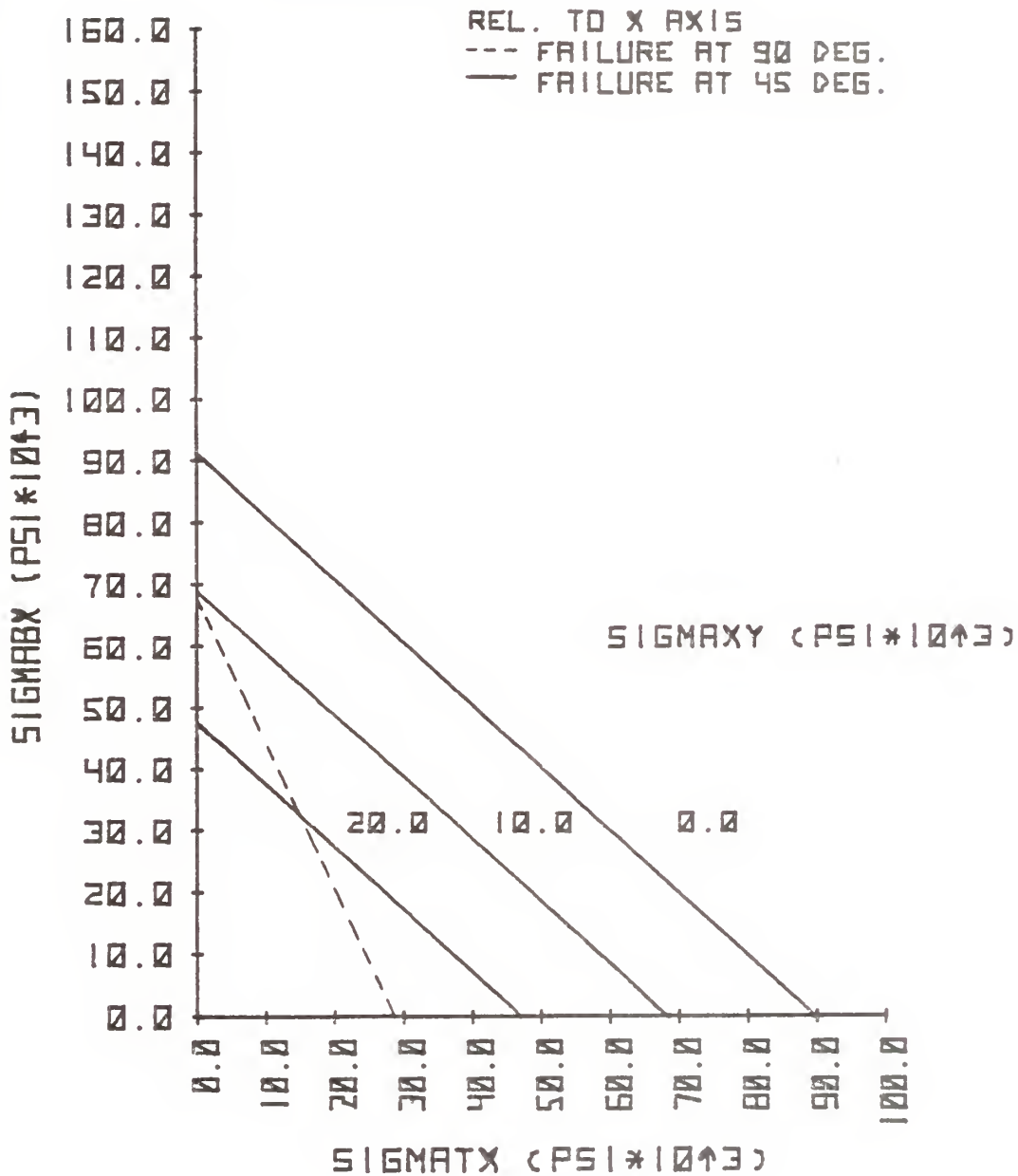


FIGURE 6. ULTIMATE STRESS INTERACTION CURVE FOR A  
ONE IN. SQUARE PLATE OF NARMCO 5209/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 10  
PER CENT ZERO DEGREE PLIES

0.25 IN. DIAM. HOLE  
20% ZERO DEG. PLIES

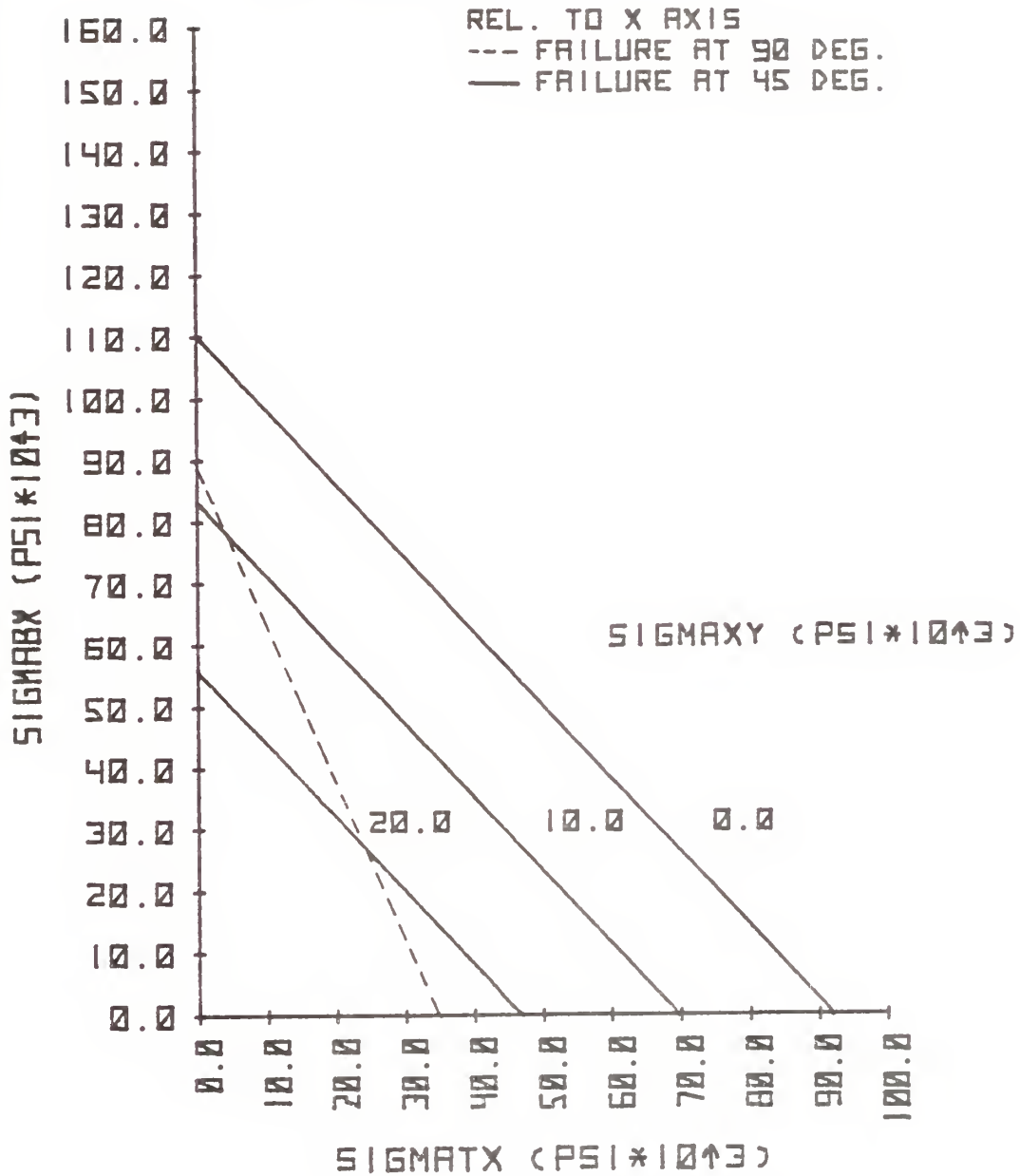


FIGURE 7. ULTIMATE STRESS INTERACTION CURVE FOR A  
ONE IN. SQUARE PLATE OF NARMCO 5203/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 20  
PER CENT ZERO DEGREE PLIES

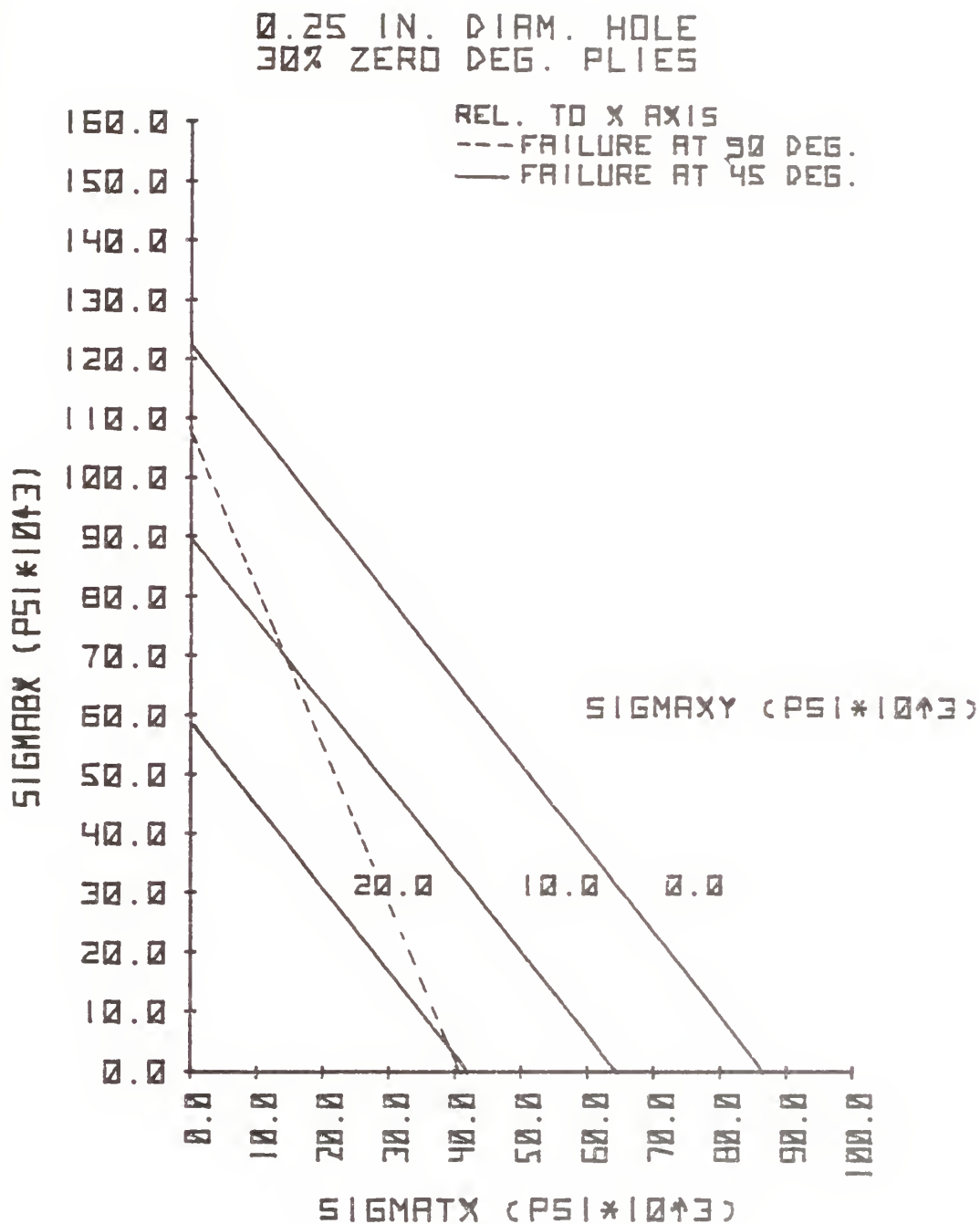


FIGURE 3. ULTIMATE STRESS INTERACTION CURVE FOR A  
ONE IN. SQUARE PLATE OF NARMCO 5203/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 30  
PER CENT ZERO DEGREE PLIES

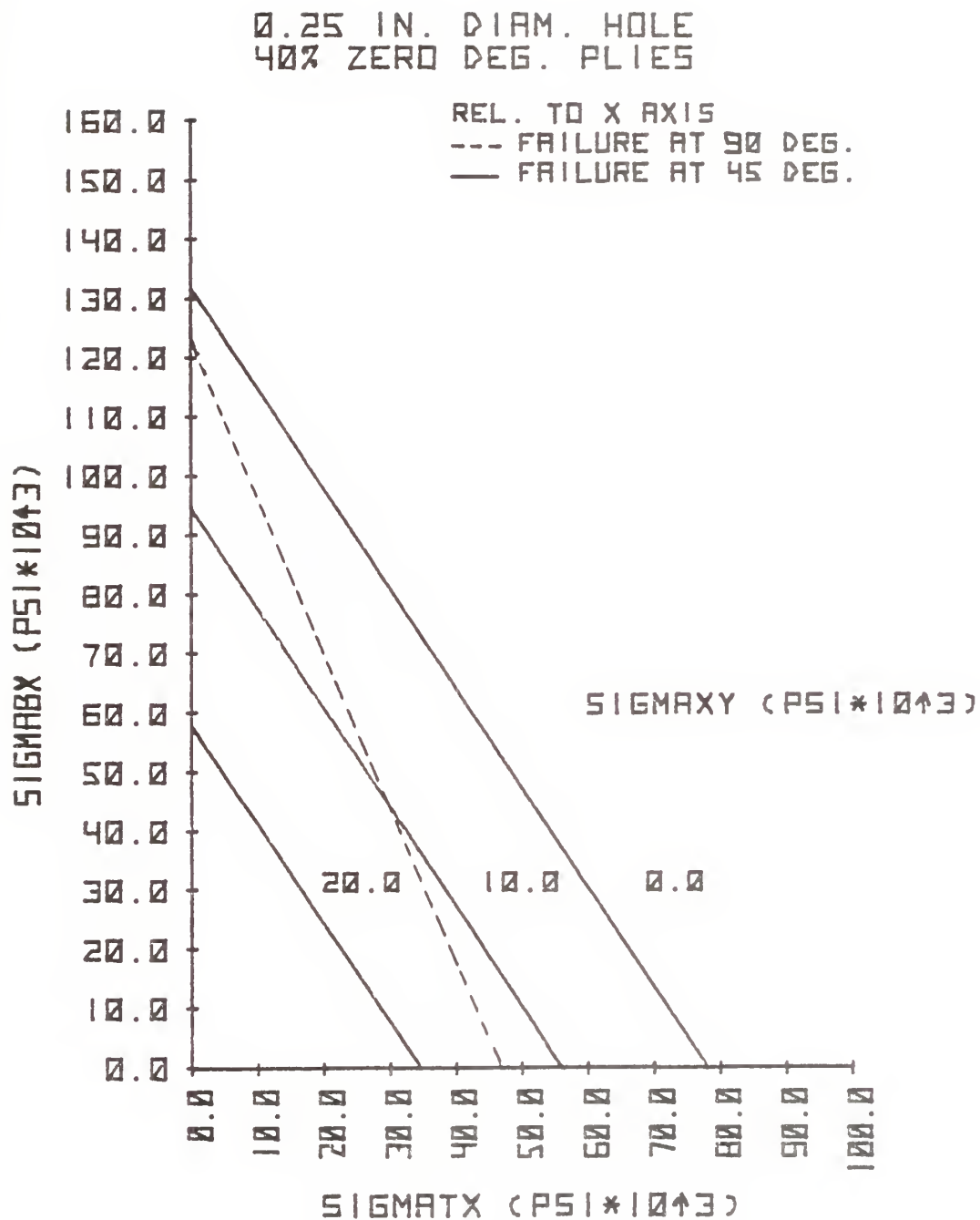


FIGURE 9. ULTIMATE STRESS INTERACTION CURVE FOR A  
ONE IN. SQUARE PLATE OF NARMCO 5208/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 40  
PER CENT ZERO DEGREE PLIES



0.25 IN. DIAM. HOLE  
50% ZERO DEG. PLIES

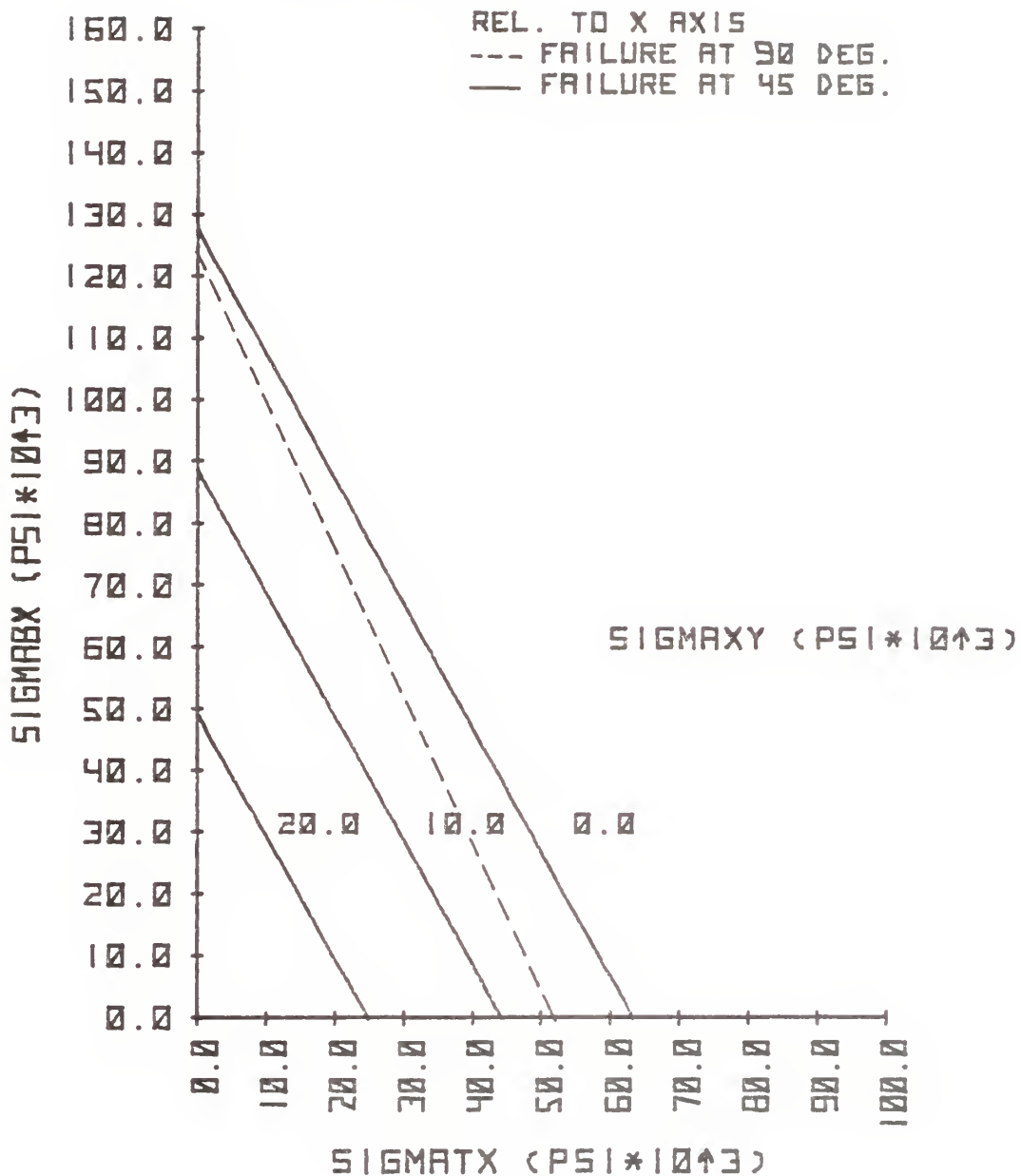


FIGURE 10. ULTIMATE STRESS INTERACTION CURVE FOR A  
ONE IN. SQUARE PLATE OF NARMCO 5203/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 50  
PER CENT ZERO DEGREE PLIES

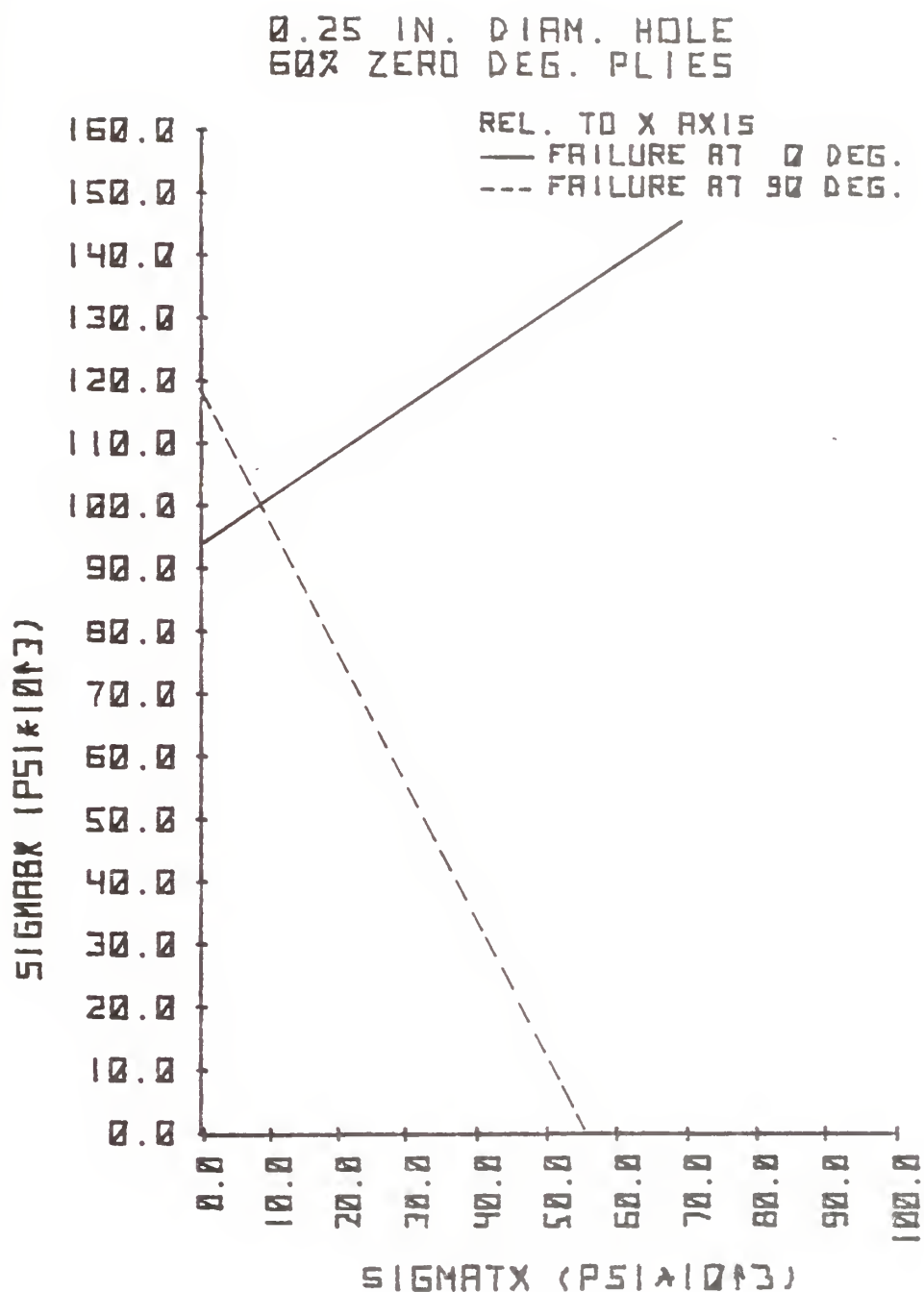


FIGURE 11. ULTIMATE STRESS INTERACTION CURVE FOR A  
ONE IN. SQUARE PLATE OF NARMCO 5208/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.25 IN. DIAMETER CENTRAL HOLE AND 60  
PER CENT ZERO DEGREE PLIES

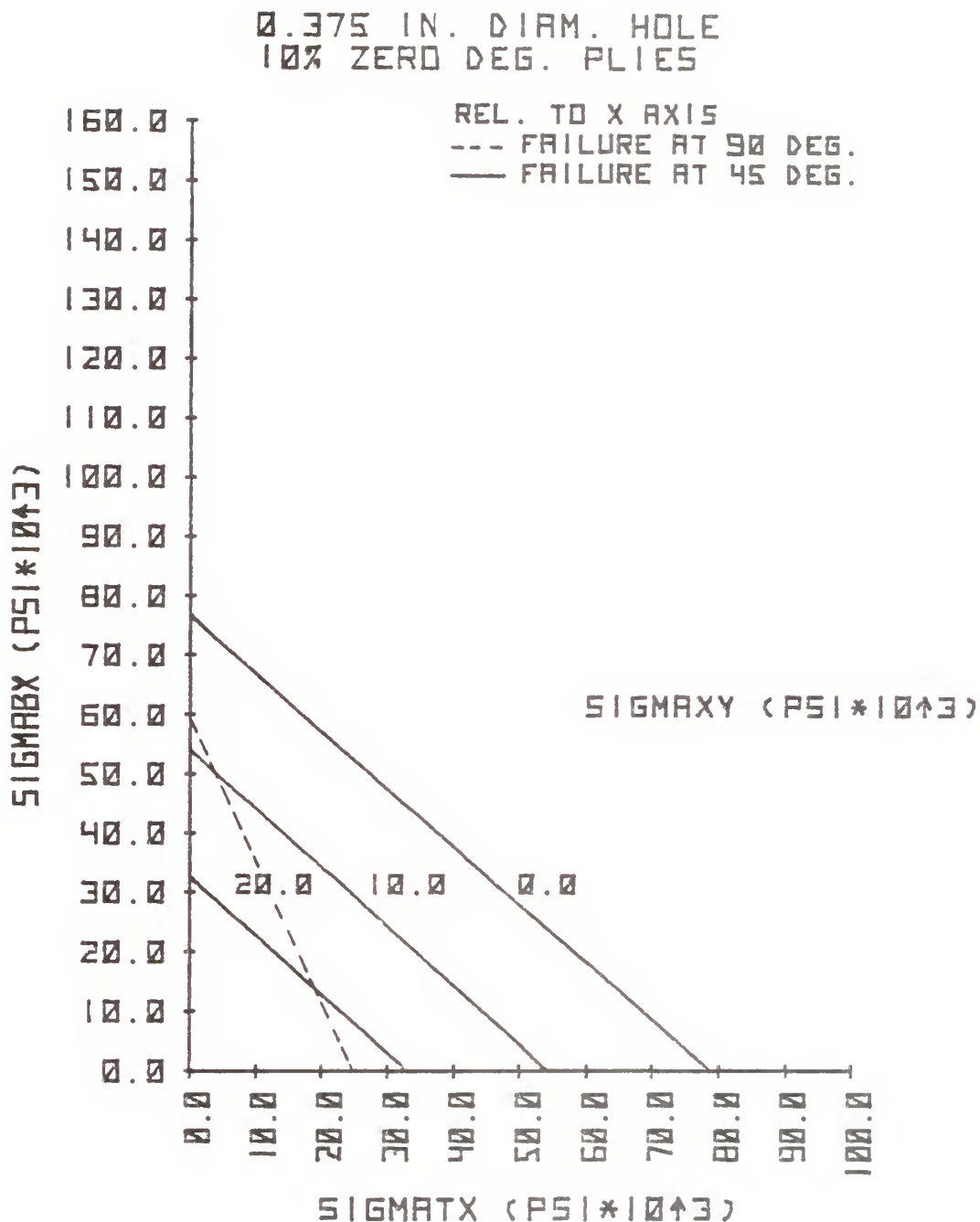


FIGURE 12. ULTIMATE STRESS INTERACTION CURVE FOR A  
1.5 IN. SQUARE PLATE OF NARMCO 5208/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND  
10 PER CENT ZERO DEGREE PLIES

0.375 IN. DIAM. HOLE  
20% ZERO DEG. PLIES

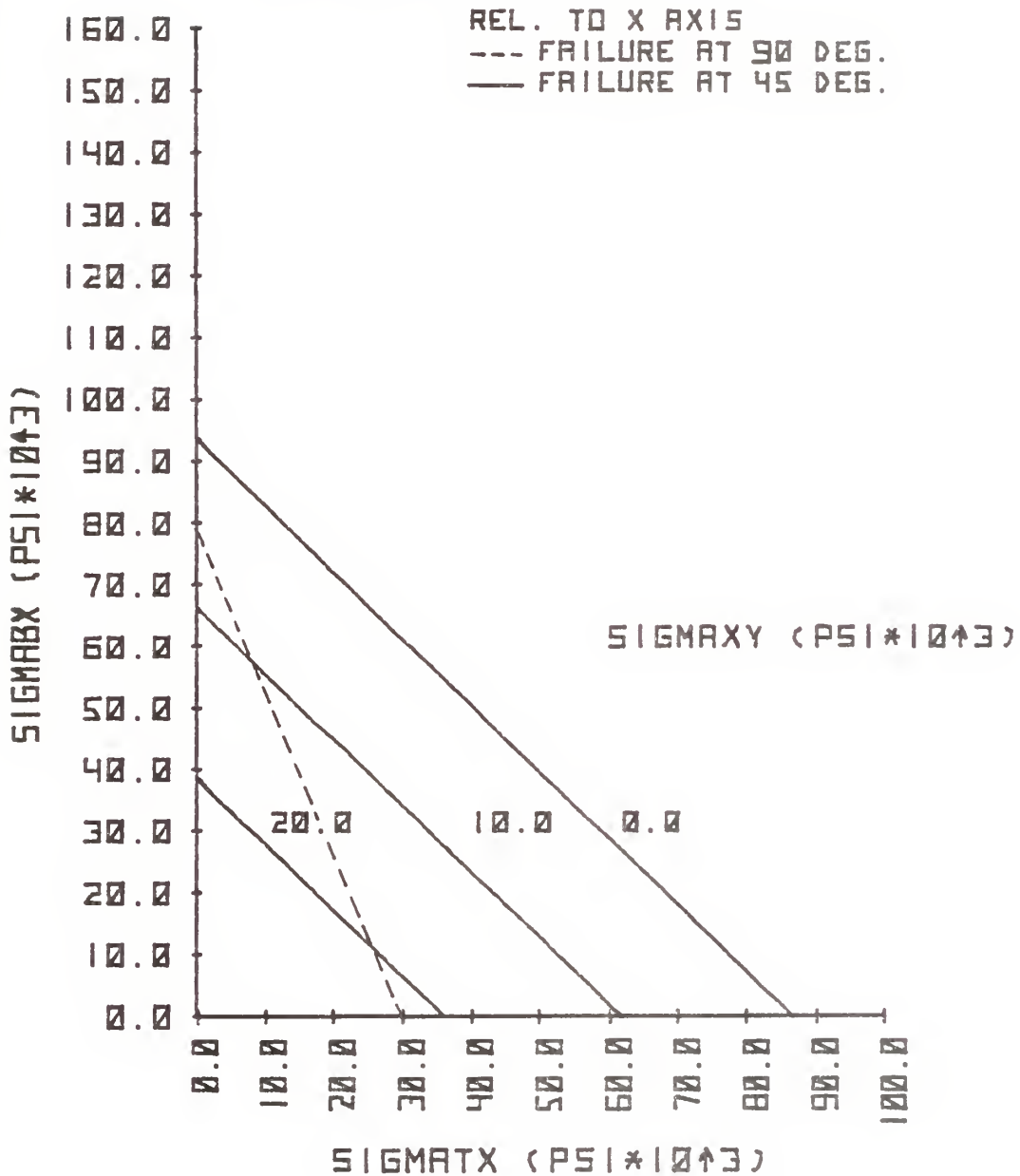


FIGURE 13. ULTIMATE STRESS INTERACTION CURVE FOR A  
1.5 IN. SQUARE PLATE OF NARMCO 5208/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND  
20 PER CENT ZERO DEGREE PLIES

0.375 IN. DIAM. HOLE  
30% ZERO DEG. PLIES

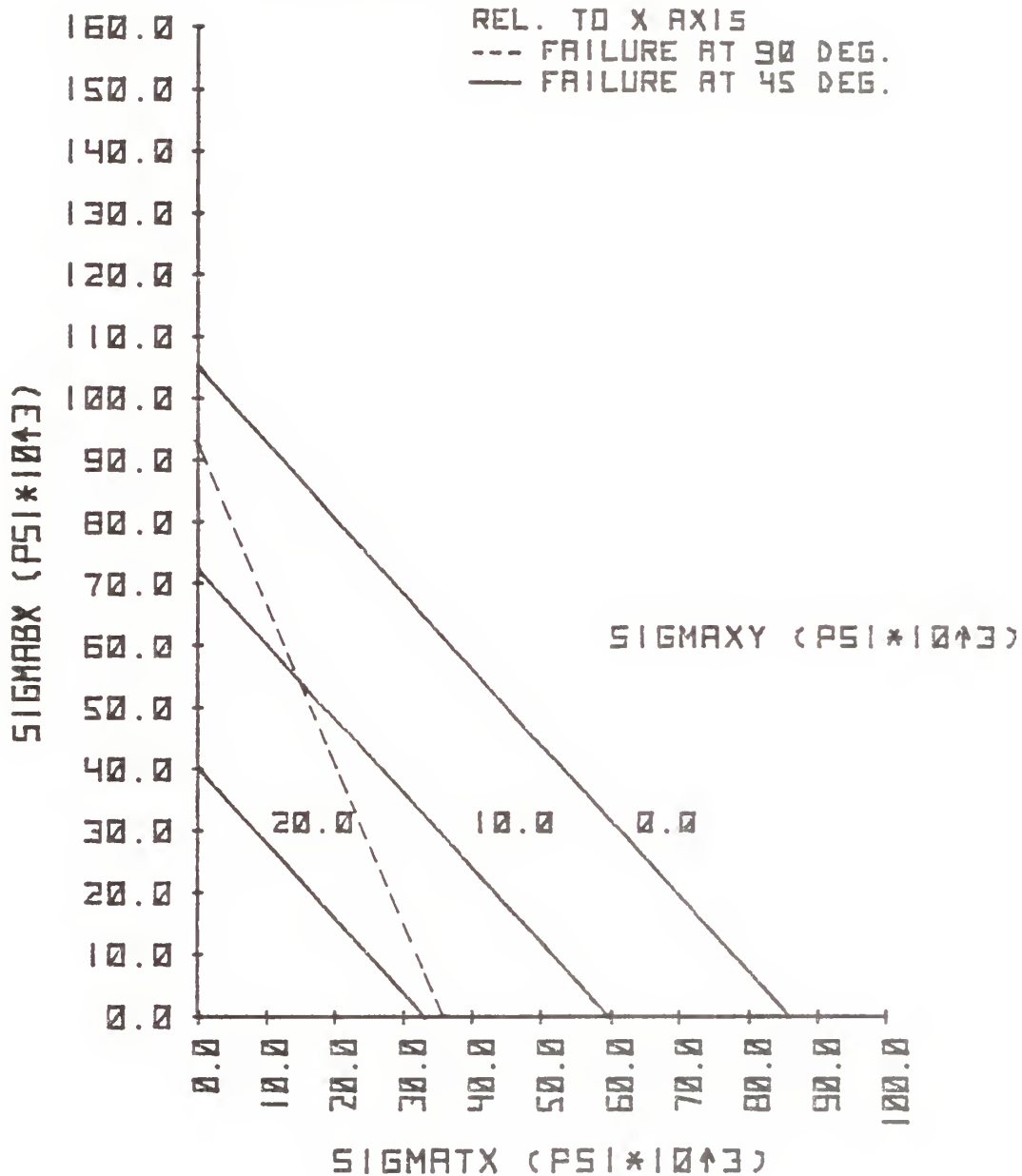


FIGURE 14. ULTIMATE STRESS INTERACTION CURVE FOR A  
1.5 IN. SQUARE PLATE OF NARMCO 5208/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND  
30 PER CENT ZERO DEGREE PLIES

0.375 IN. DIAM. HOLE  
40% ZERO DEG. PLIES

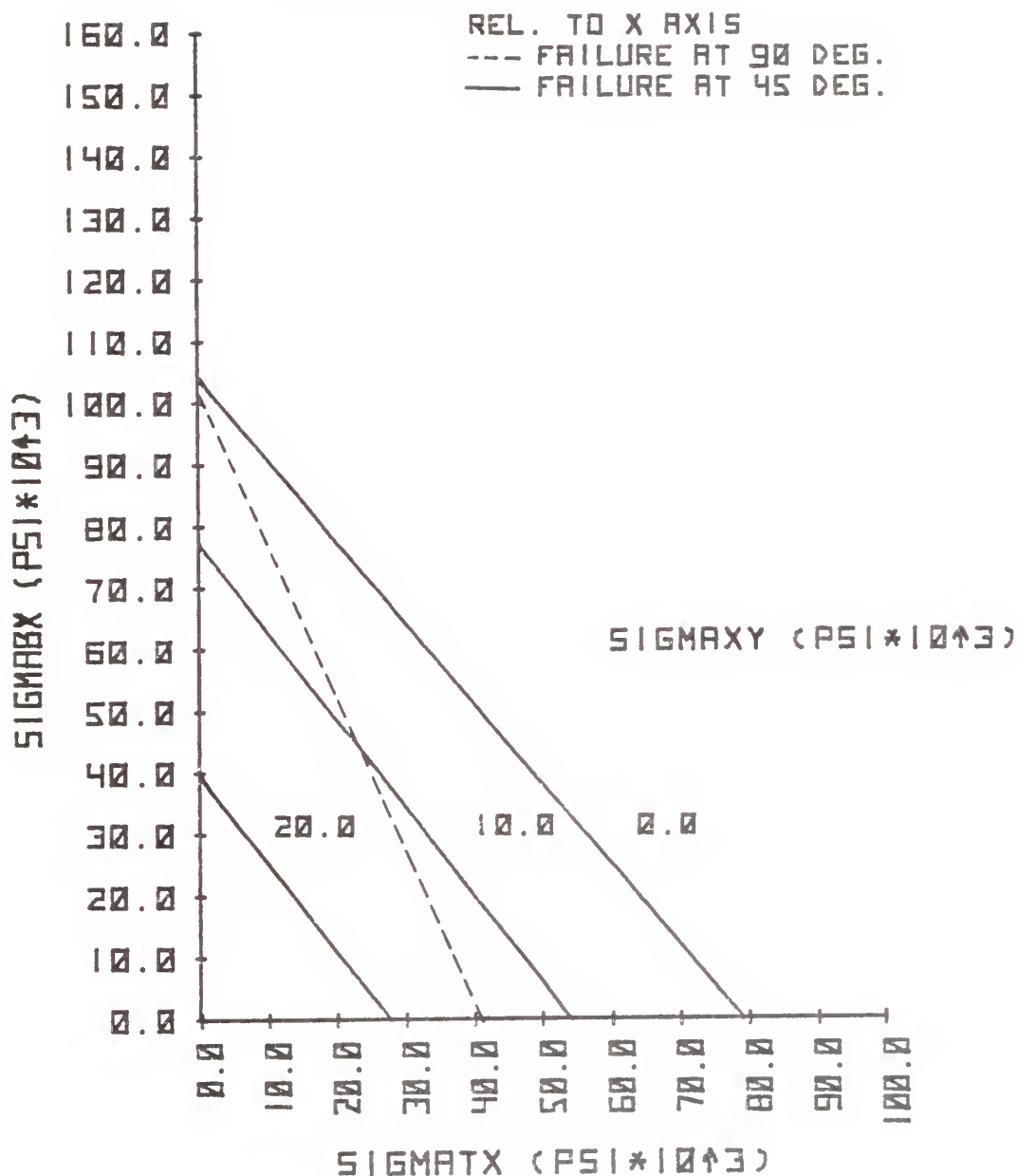


FIGURE 15. ULTIMATE STRESS INTERACTION CURVE FOR A  
1.5 IN. SQUARE PLATE OF NARMCO 5208/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND  
40 PER CENT ZERO DEGREE PLIES

0.375 IN. DIAM. HOLE  
50% ZERO DEG. PLIES

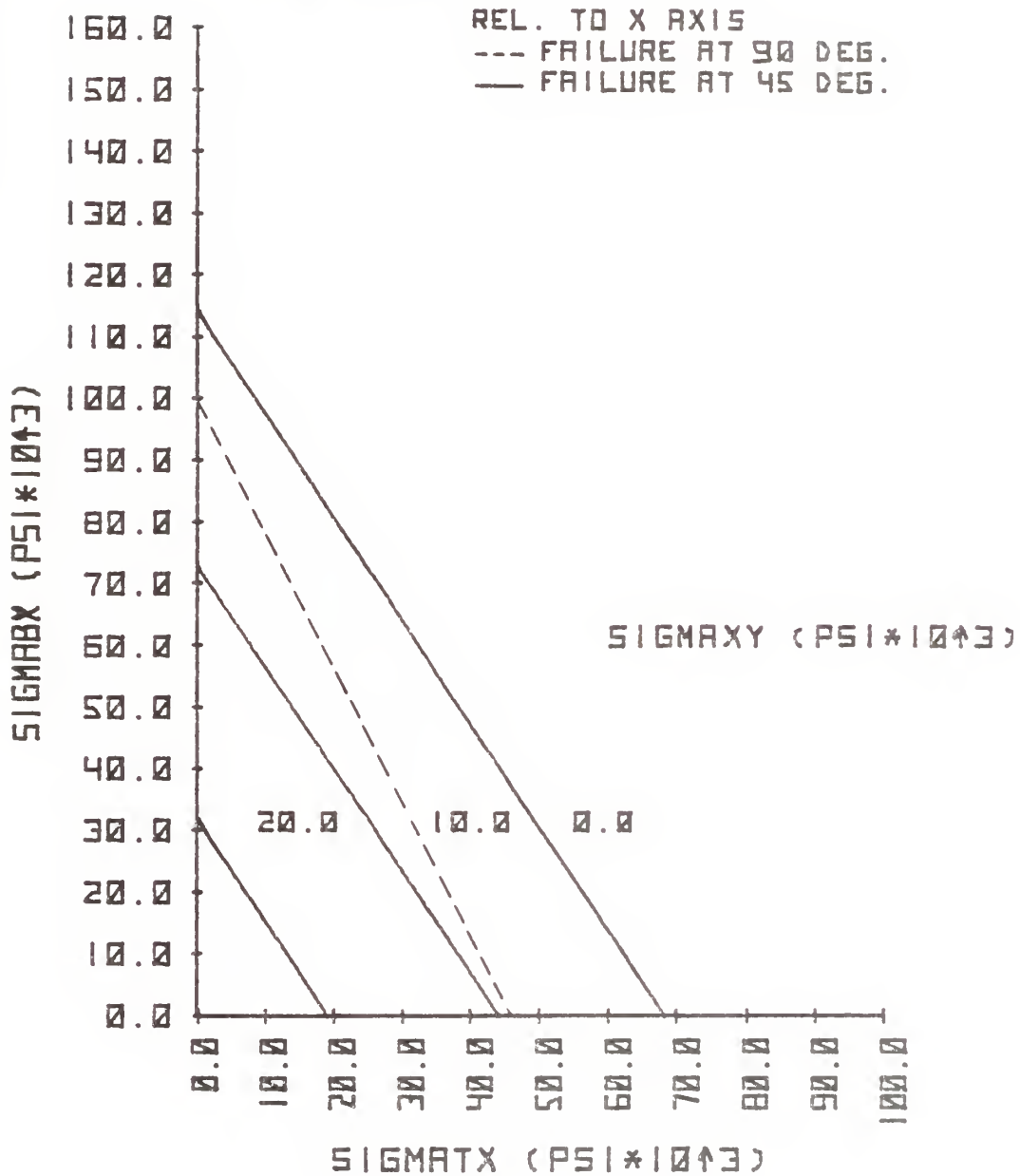


FIGURE 16. ULTIMATE STRESS INTERACTION CURVE FOR A  
1.5 IN. SQUARE PLATE OF NARMCO 5203/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND  
50 PER CENT ZERO DEGREE PLIES

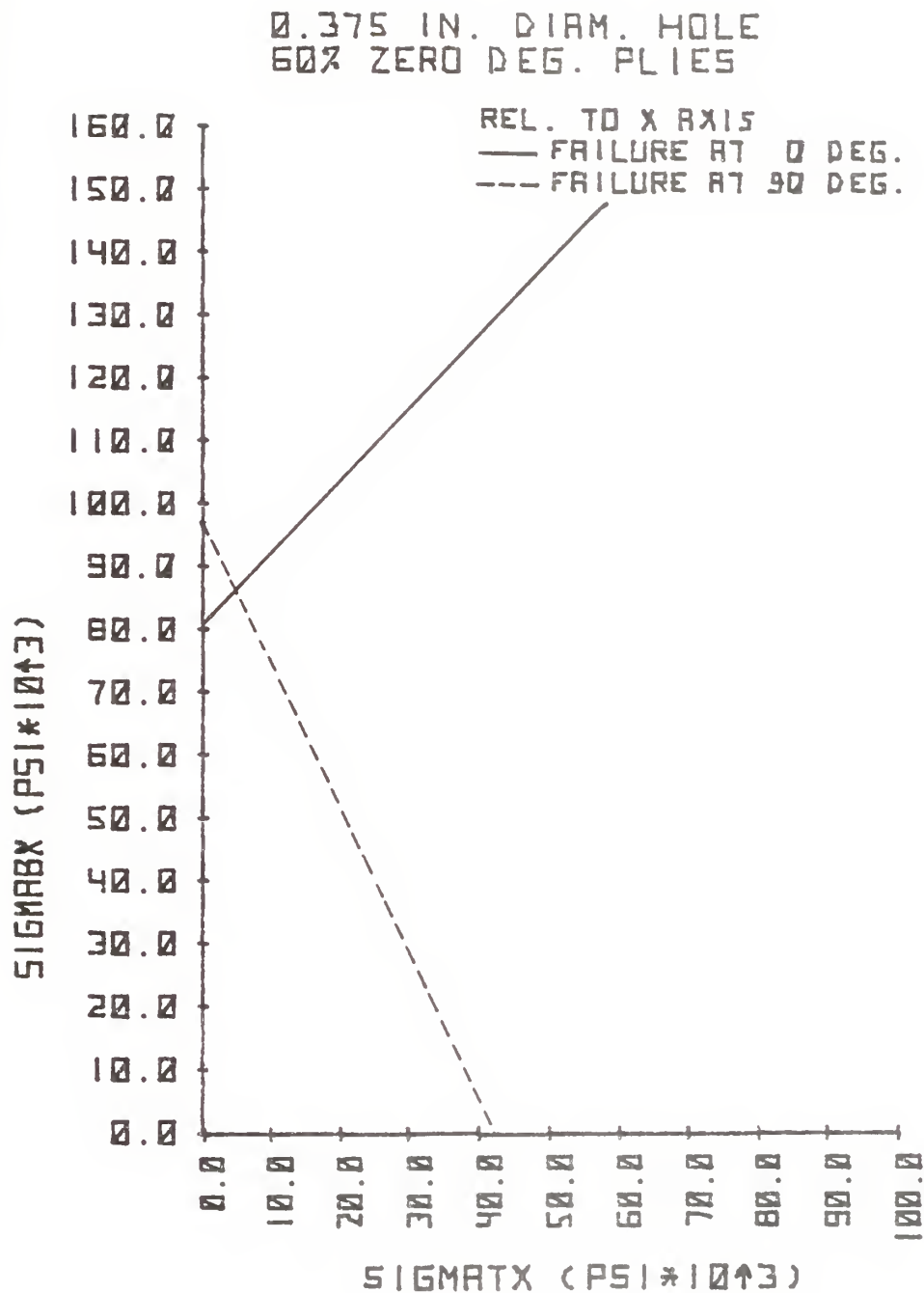


FIGURE 17. ULTIMATE STRESS INTERACTION CURVE FOR A  
1.5 IN. SQUARE PLATE OF NARMCO 5208/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.375 IN. DIAMETER CENTRAL HOLE AND  
60 PER CENT ZERO DEGREE PLIES



0.4375 IN. DIAM. HOLE  
10% ZERO DEG. PLIES

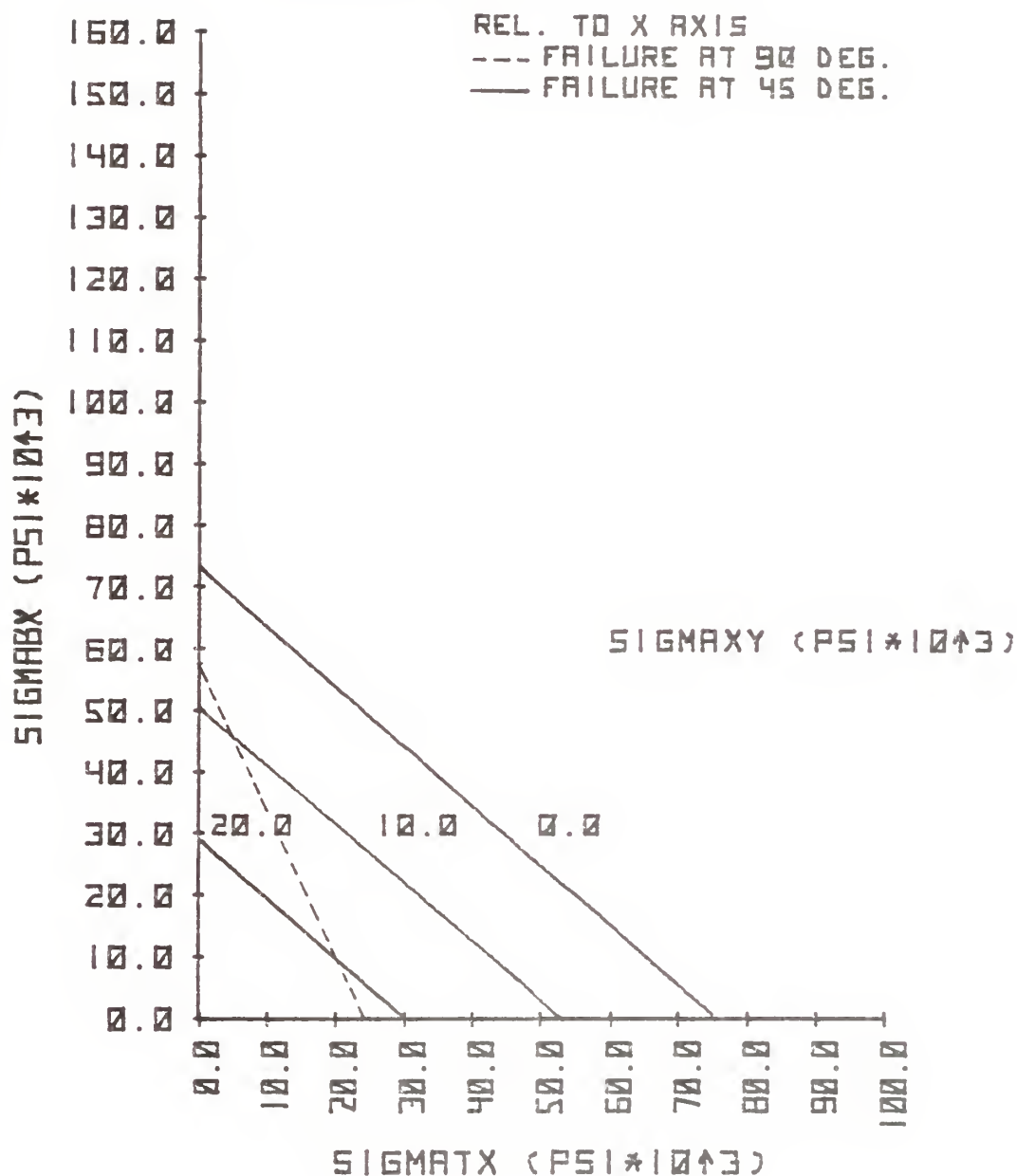


FIGURE 18. ULTIMATE STRESS INTERACTION CURVE FOR A  
1.75 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45]  
MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND  
10 PER CENT ZERO DEGREE PLIES

0.4375 IN. DIAM. HOLE  
20% ZERO DEG. PLIES

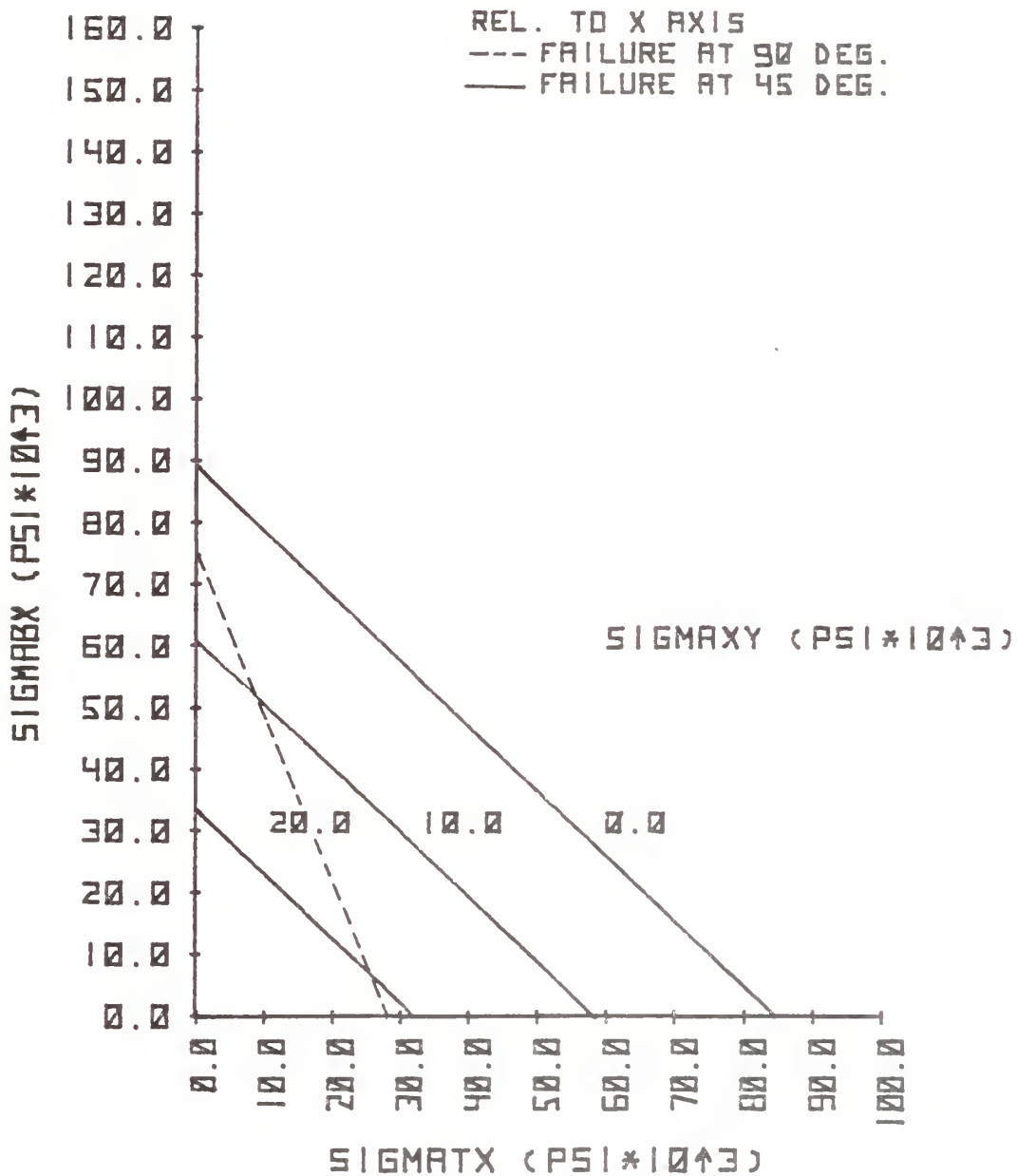


FIGURE 19. ULTIMATE STRESS INTERACTION CURVE FOR A  
1.75 IN. SQUARE PLATE OF NARMCO 5208/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND  
20 PER CENT ZERO DEGREE PLIES

0.4375 IN. DIAM. HOLE  
30% ZERO DEG. PLIES

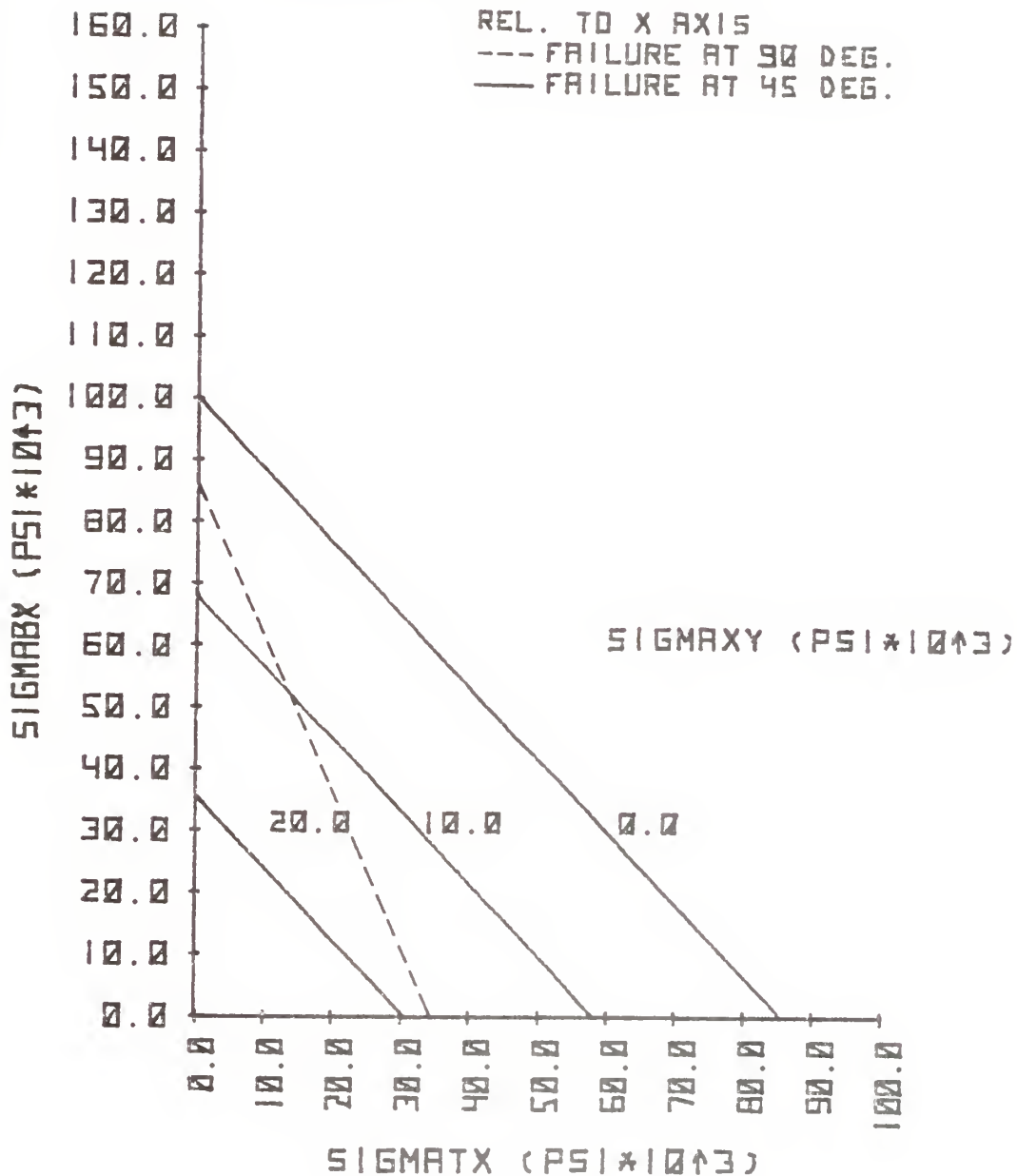


FIGURE 20. ULTIMATE STRESS INTERACTION CURVE FOR A  
1.75 IN. SQUARE PLATE OF NARMCO 5203/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND  
30 PER CENT ZERO DEGREE PLIES

0.4375 IN. DIAM. HOLE  
40% ZERO DEG. PLIES

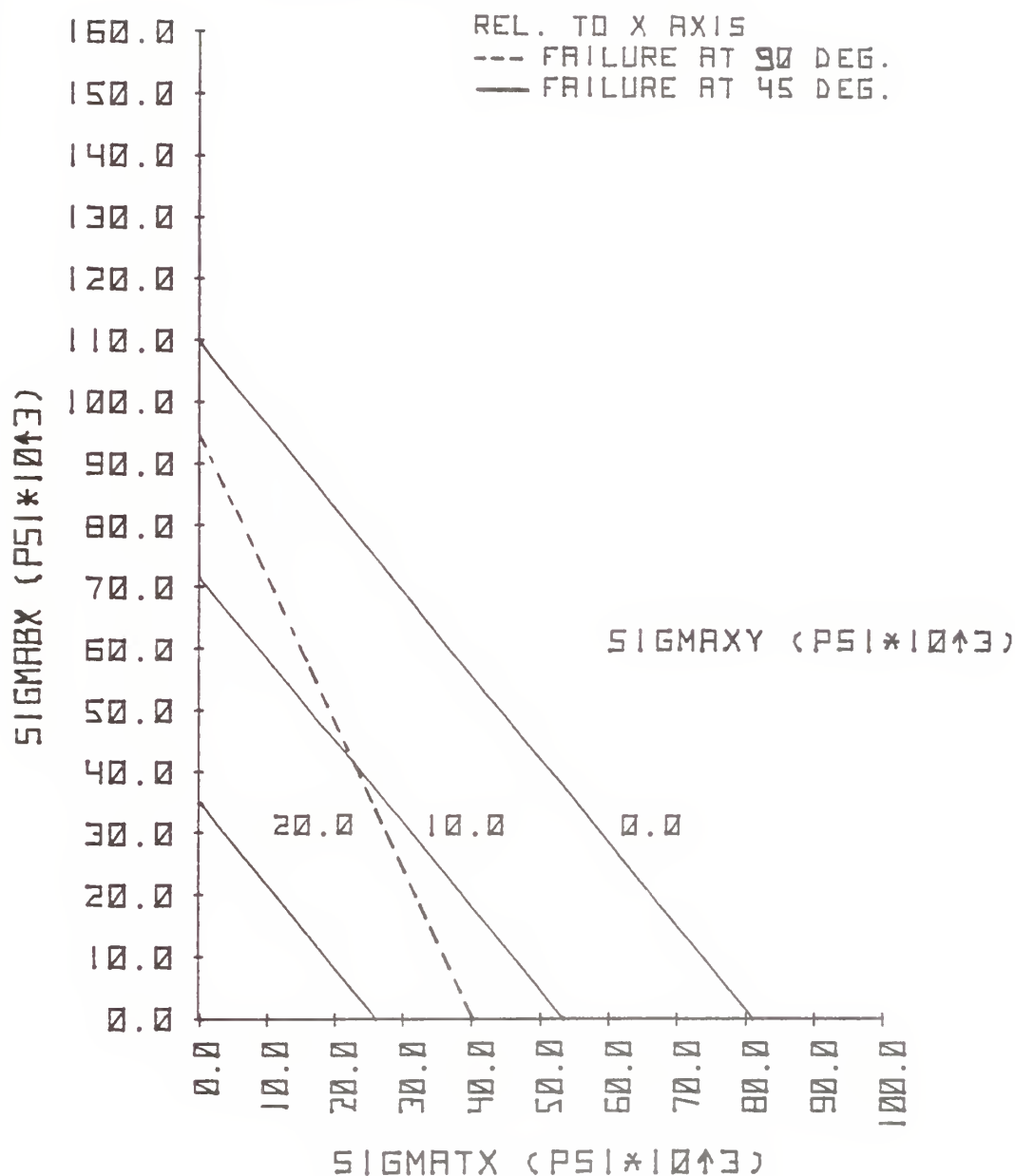


FIGURE 21. ULTIMATE STRESS INTERACTION CURVE FOR A  
1.75 IN. SQUARE PLATE OF NARMCO 5209/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND  
40 PER CENT ZERO DEGREE PLIES

0.4375 IN. DIAM. HOLE  
50% ZERO DEG. PLIES

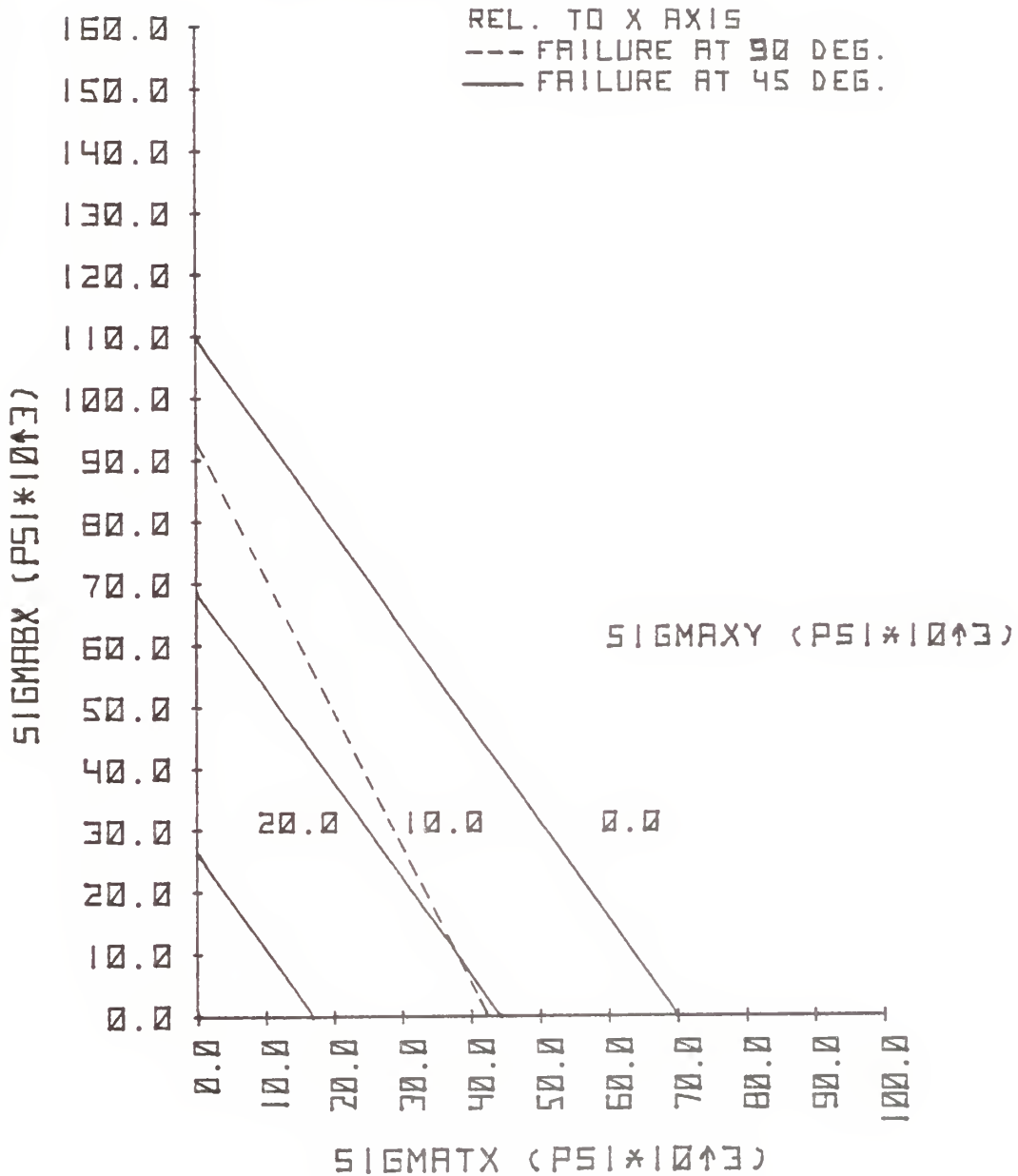


FIGURE 22. ULTIMATE STRESS INTERACTION CURVE FOR A  
1.75 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/+45]  
MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND  
50 PER CENT ZERO DEGREE PLIES

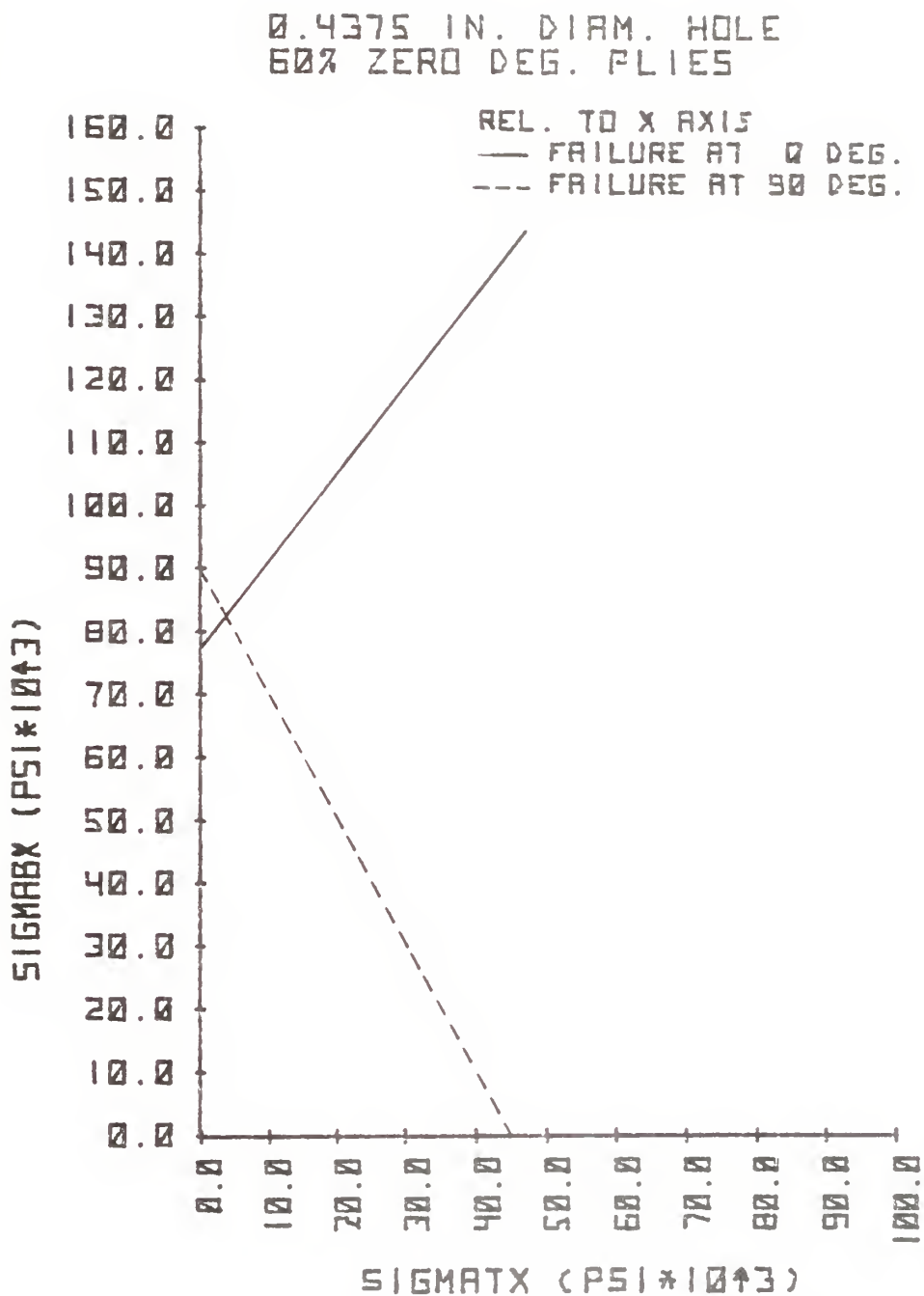


FIGURE 23. ULTIMATE STRESS INTERACTION CURVE FOR A  
1.75 IN. SQUARE PLATE OF NARMCO 5203/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.4375 IN. DIAMETER CENTRAL HOLE AND  
60 PER CENT ZERO DEGREE PLYS

0.5 IN. DIAM. HOLE  
10% ZERO DEG. PLIES

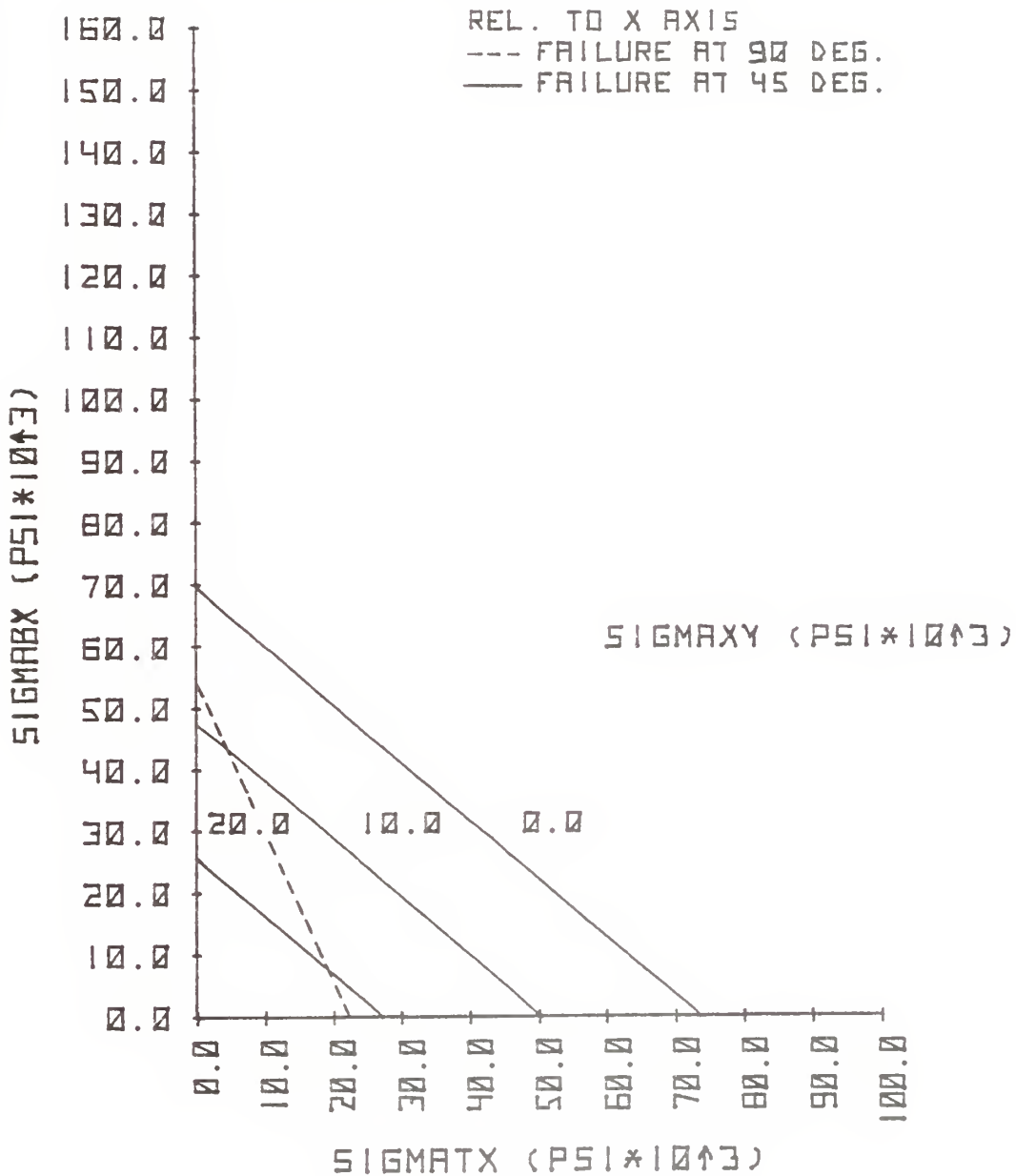


FIGURE 24. ULTIMATE STRESS INTERACTION CURVE FOR A  
2.0 IN. SQUARE PLATE OF NARMCO 5208/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 10  
PER CENT ZERO DEGREE PLIES



0.5 IN. DIAM. HOLE  
20% ZERO DEG. PLIES

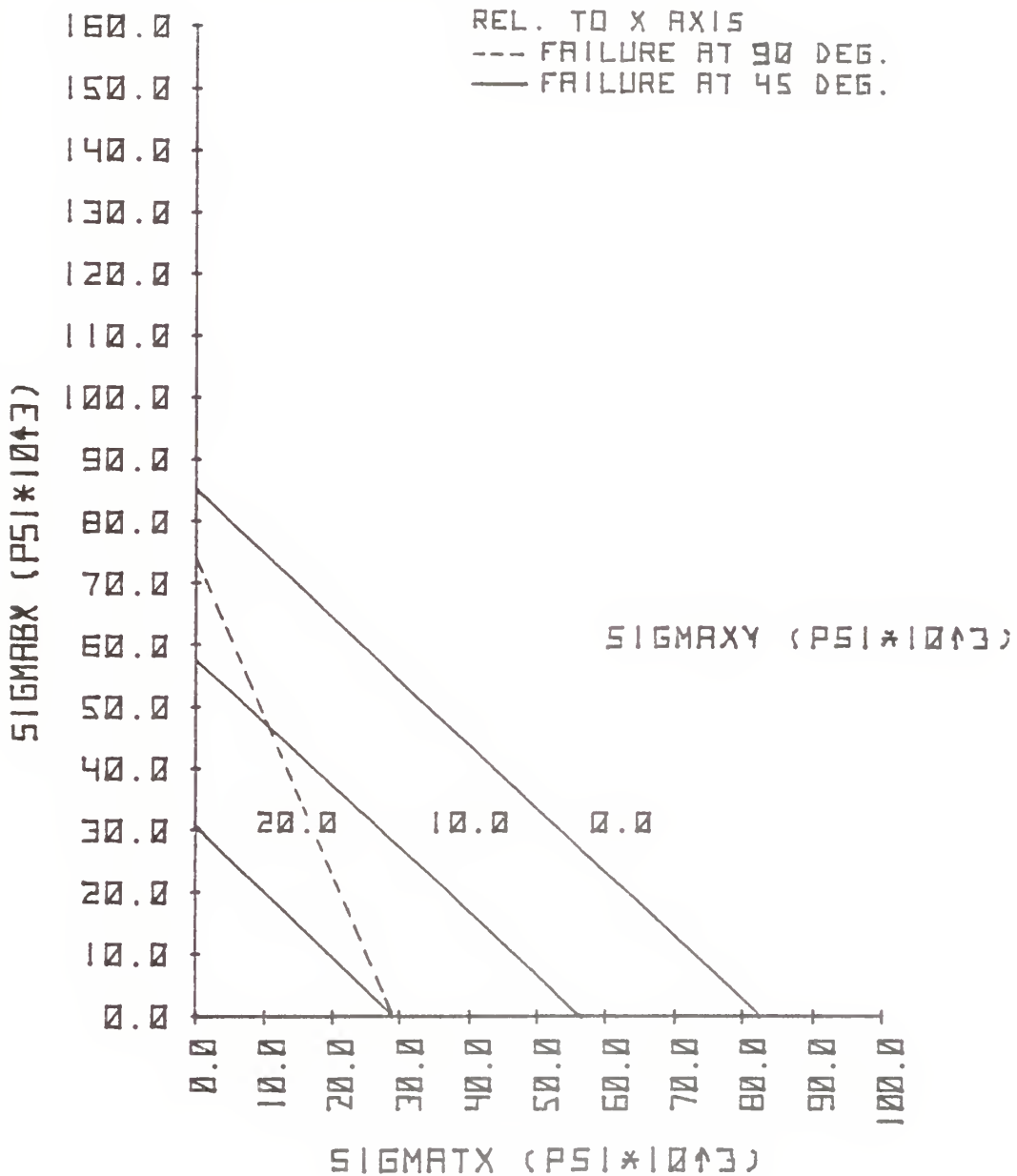


FIGURE 25. ULTIMATE STRESS INTERACTION CURVE FOR A  
2.0 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/+45]  
MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 20  
PER CENT ZERO DEGREE PLIES

0.5 IN. DIAM. HOLE  
30% ZERO DEG. PLIES

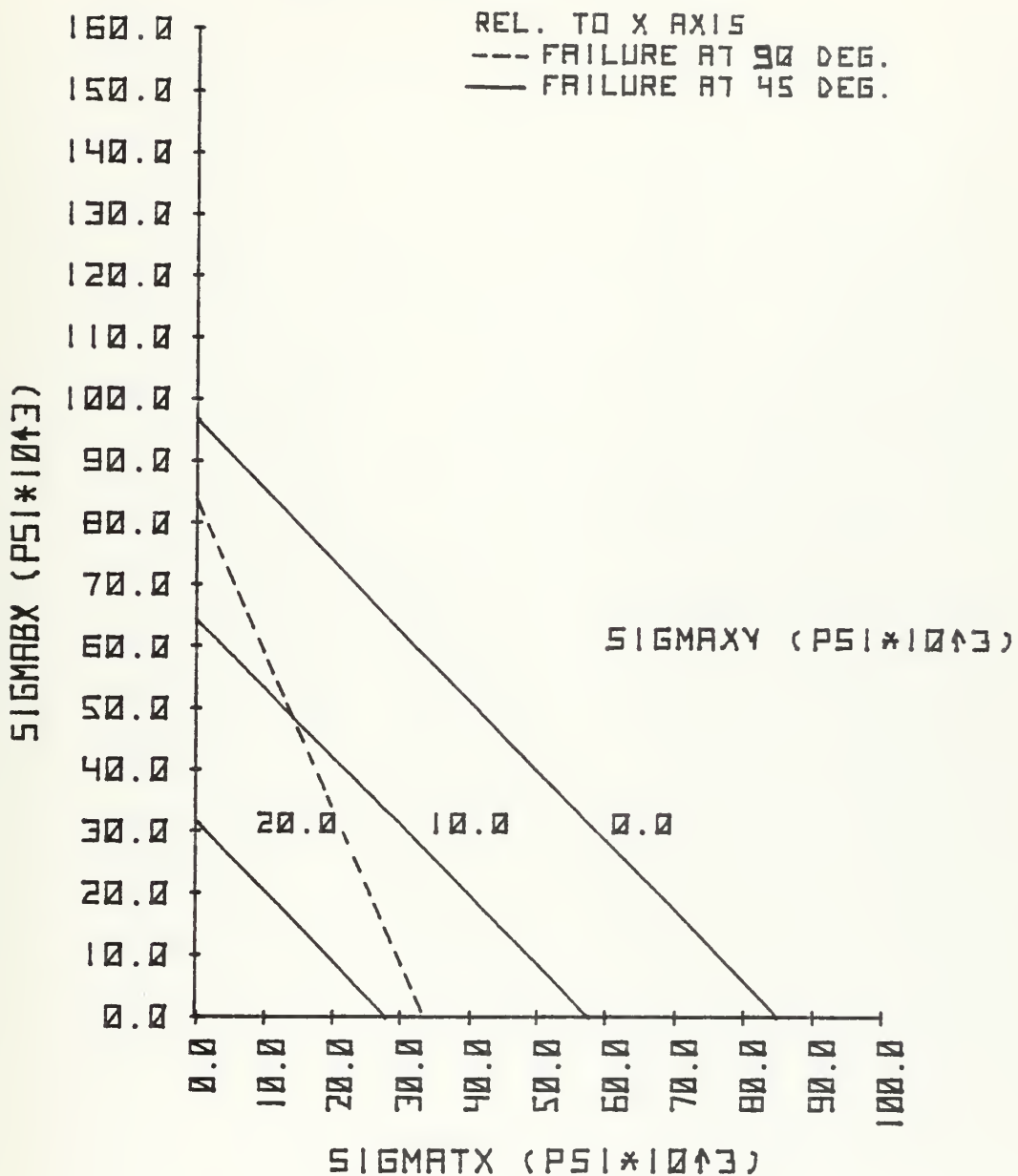


FIGURE 26. ULTIMATE STRESS INTERACTION CURVE FOR A  
2.0 IN. SQUARE PLATE OF NARMCO 5208/T300 [0/±45]  
MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 30  
PER CENT ZERO DEGREE PLIES

0.5 IN. DIAM. HOLE  
40% ZERO DEG. PLIES

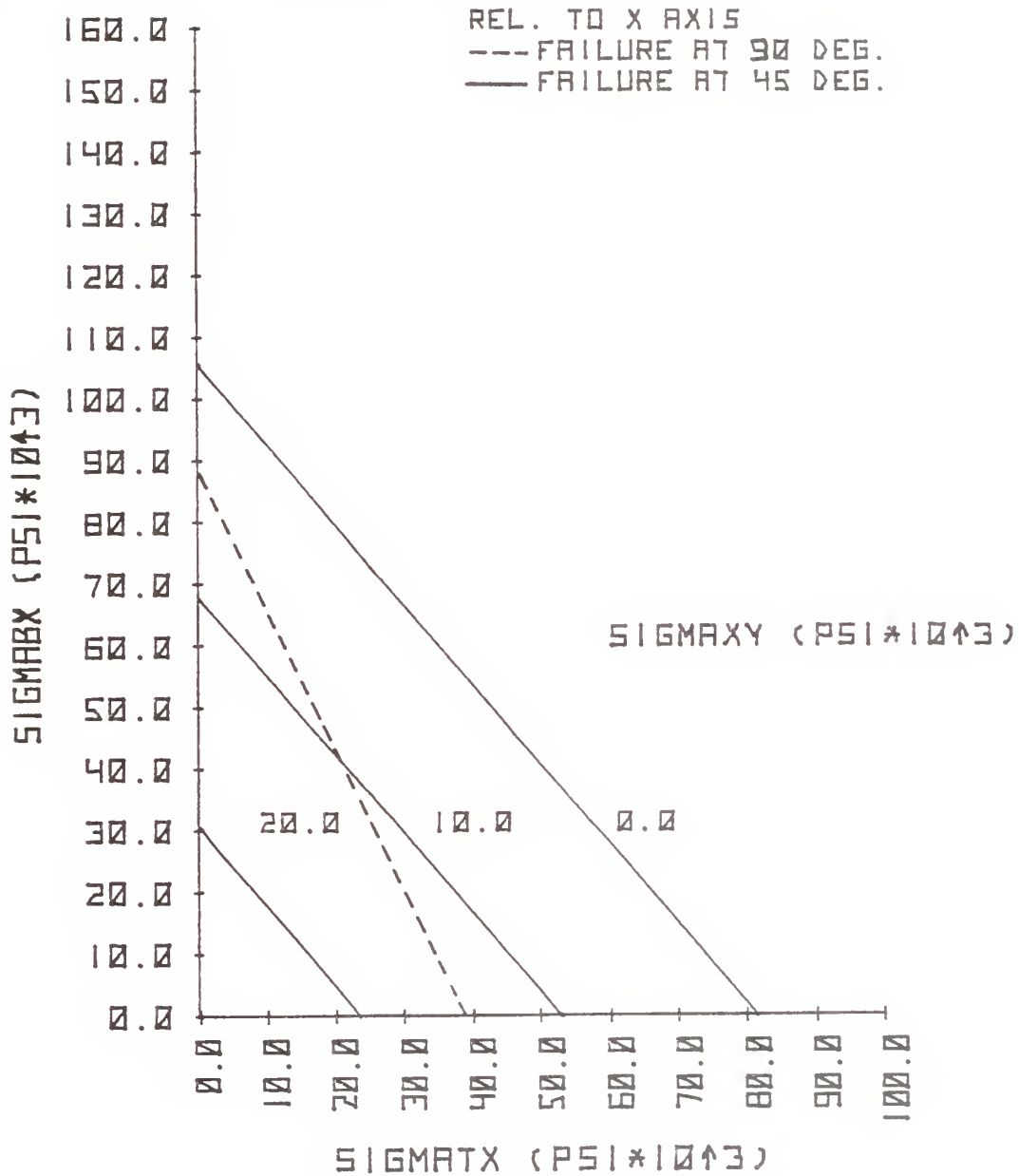


FIGURE 27. ULTIMATE STRESS INTERACTION CURVE FOR A  
2.0 IN. SQUARE PLATE OF NARMCO 5203/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 40  
PER CENT ZERO DEGREE PLIES

0.5 IN. DIAM. HOLE  
50% ZERO DEG. PLIES

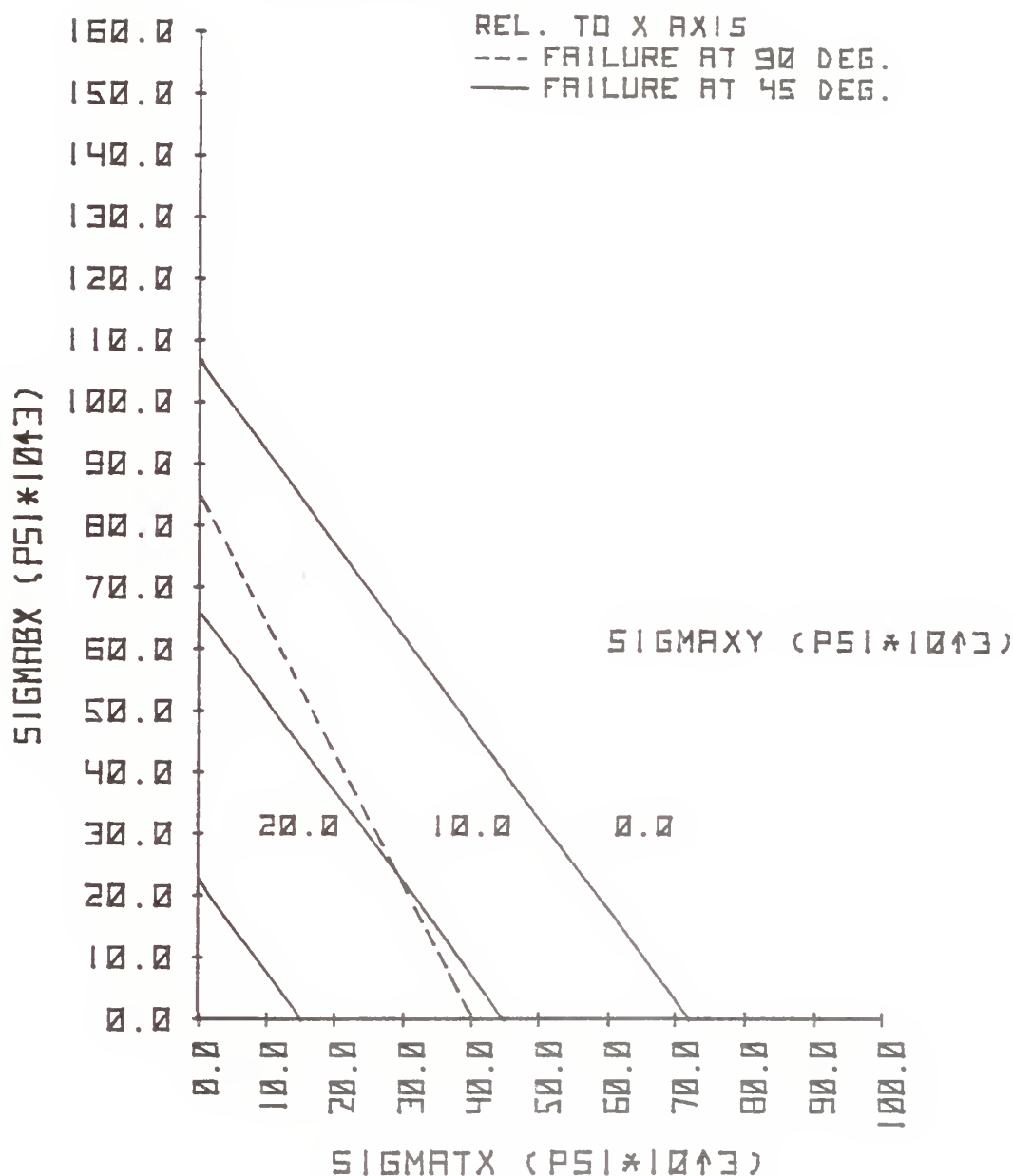


FIGURE 28. ULTIMATE STRESS INTERACTION CURVE FOR A  
2.0 IN. SQUARE PLATE OF NARMCO 5208/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 50  
PER CENT ZERO DEGREE PLIES

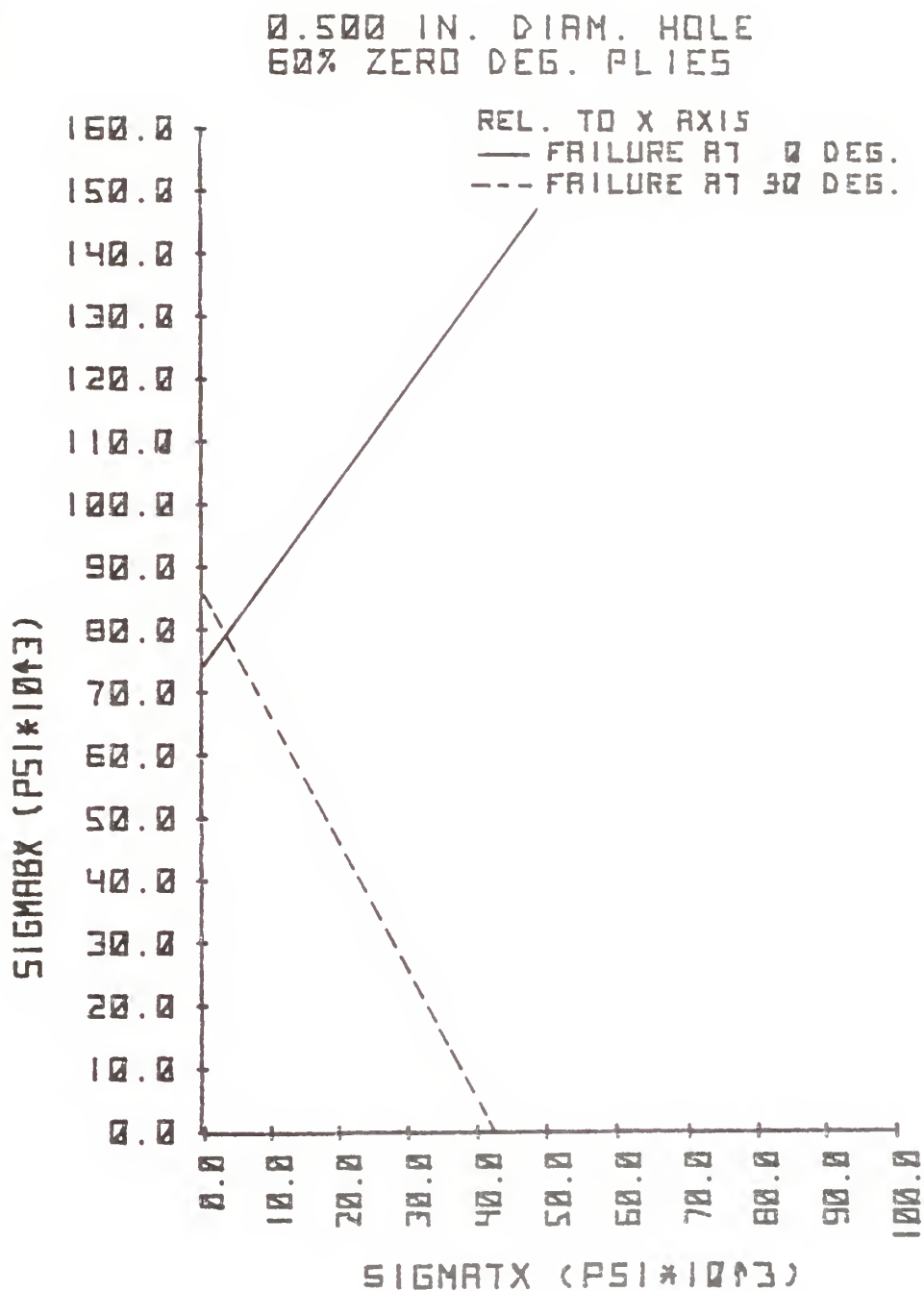


FIGURE 29. ULTIMATE STRESS INTERACTION CURVE FOR A  
2.0 IN. SQUARE PLATE OF NARMCO 5208/T300  $[0/\pm 45]$   
MATERIAL WITH A 0.5 IN. DIAMETER CENTRAL HOLE AND 60  
PER CENT ZERO DEGREE PLIES

# EXCESS BEARING CAPACITY CALCULATIONS

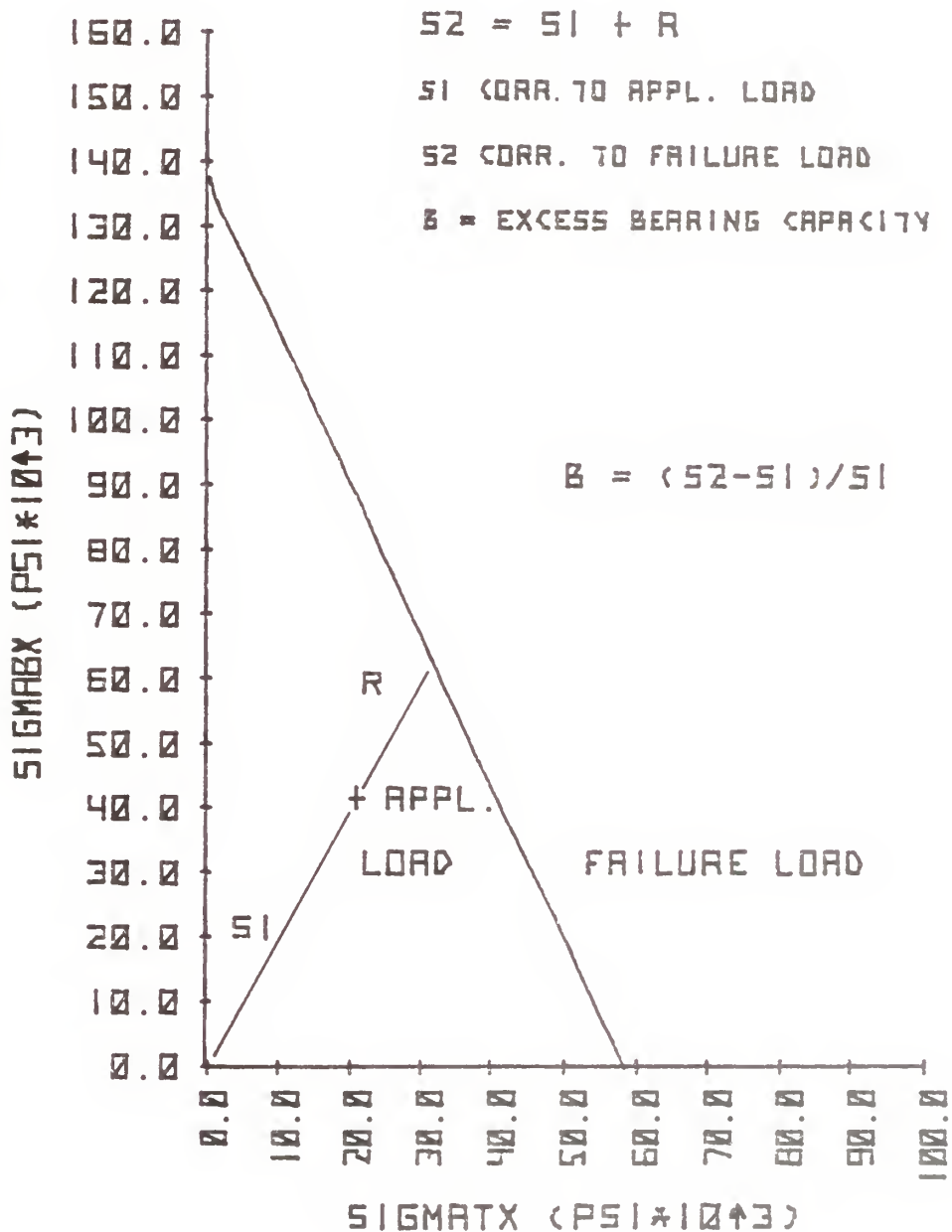
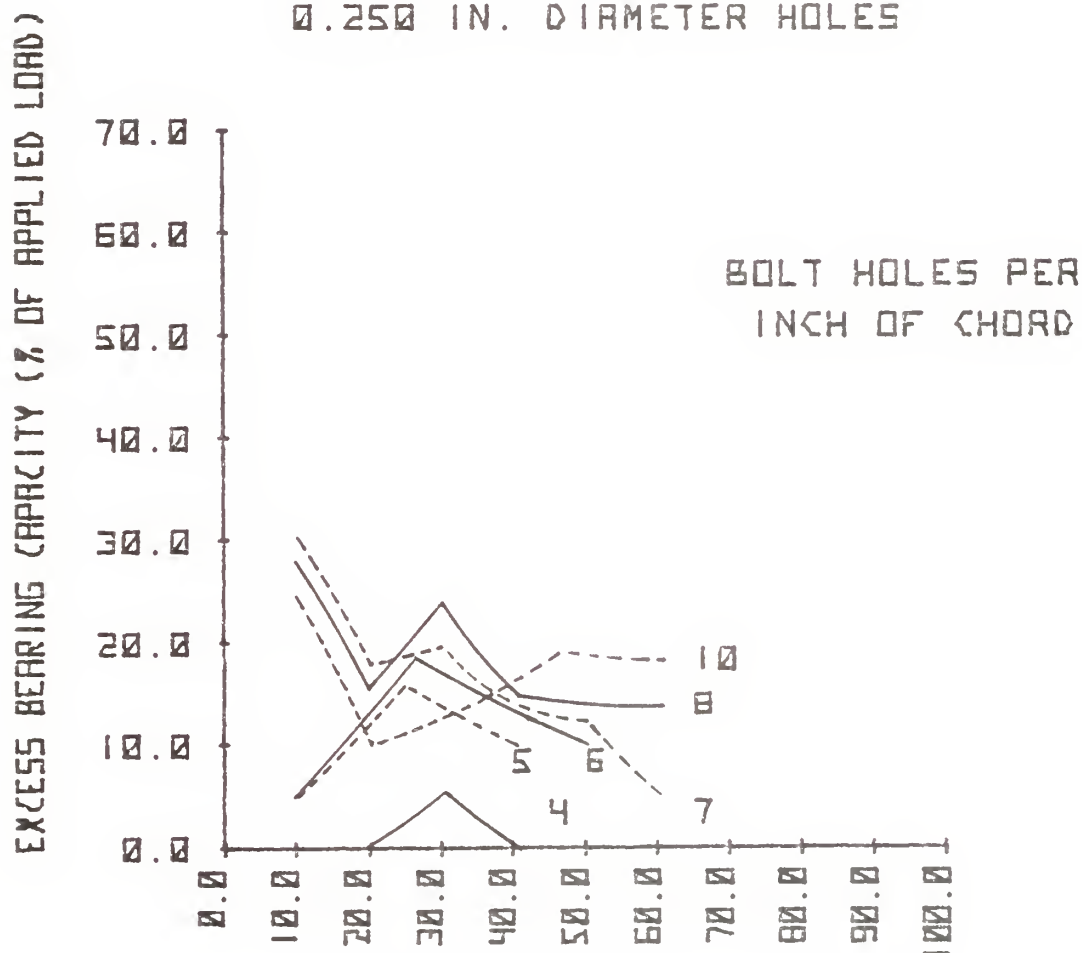


FIGURE 30. EXCESS BEARING CAPACITY CALCULATIONS

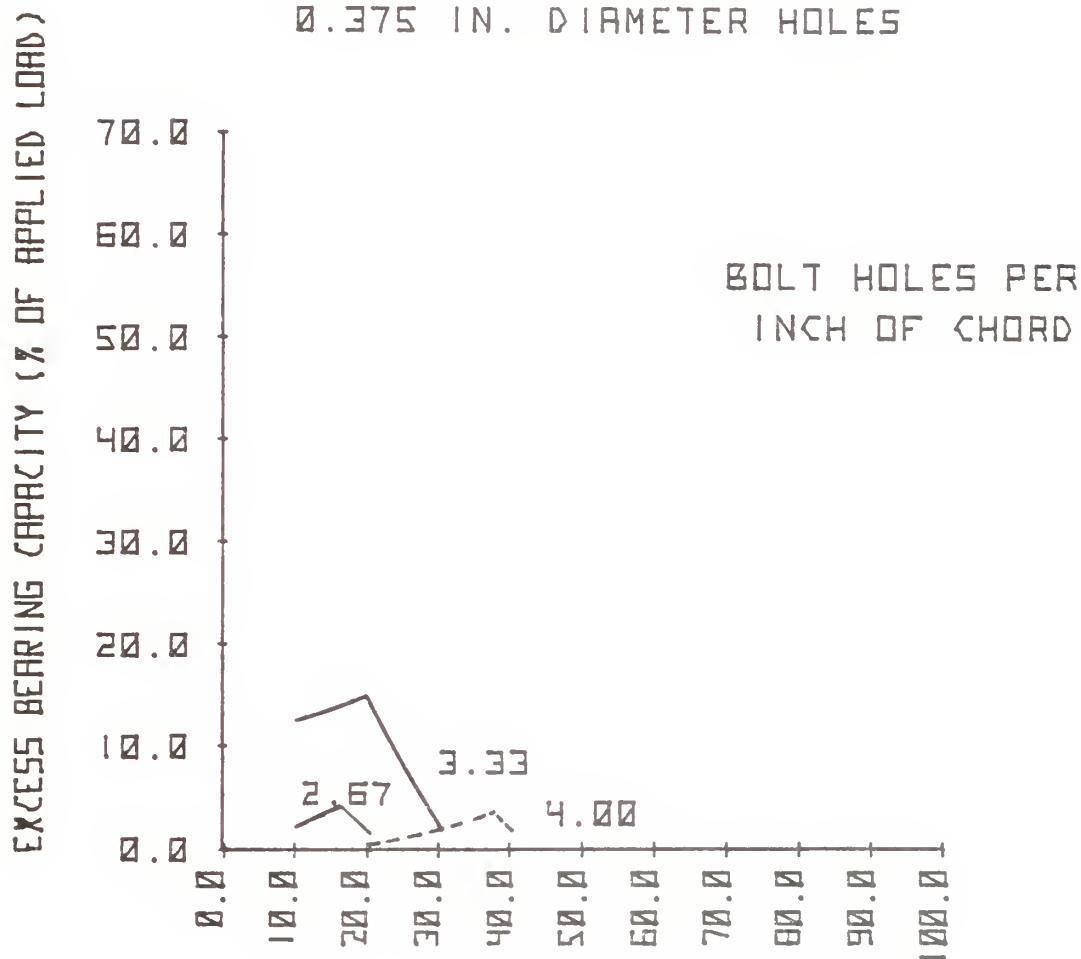
VARIATION OF EXCESS BEARING CAPACITY  
WITH LAMINATE COMPOSITION  
NON-BUFFER STRIP JOINT  
0.250 IN. DIAMETER HOLES



PER CENT ZERO DEG. PLYS AT OUTBOARD HOLE

FIGURE 31. VARIATION OF EXCESS BEARING CAPACITY  
WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP  
JOINTS WITH 0.25 IN. DIAMETER BOLT HOLES

VARIATION OF EXCESS BEARING CAPACITY  
WITH LAMINATE COMPOSITION  
NON-BUFFER STRIP JOINT  
0.375 IN. DIAMETER HOLES

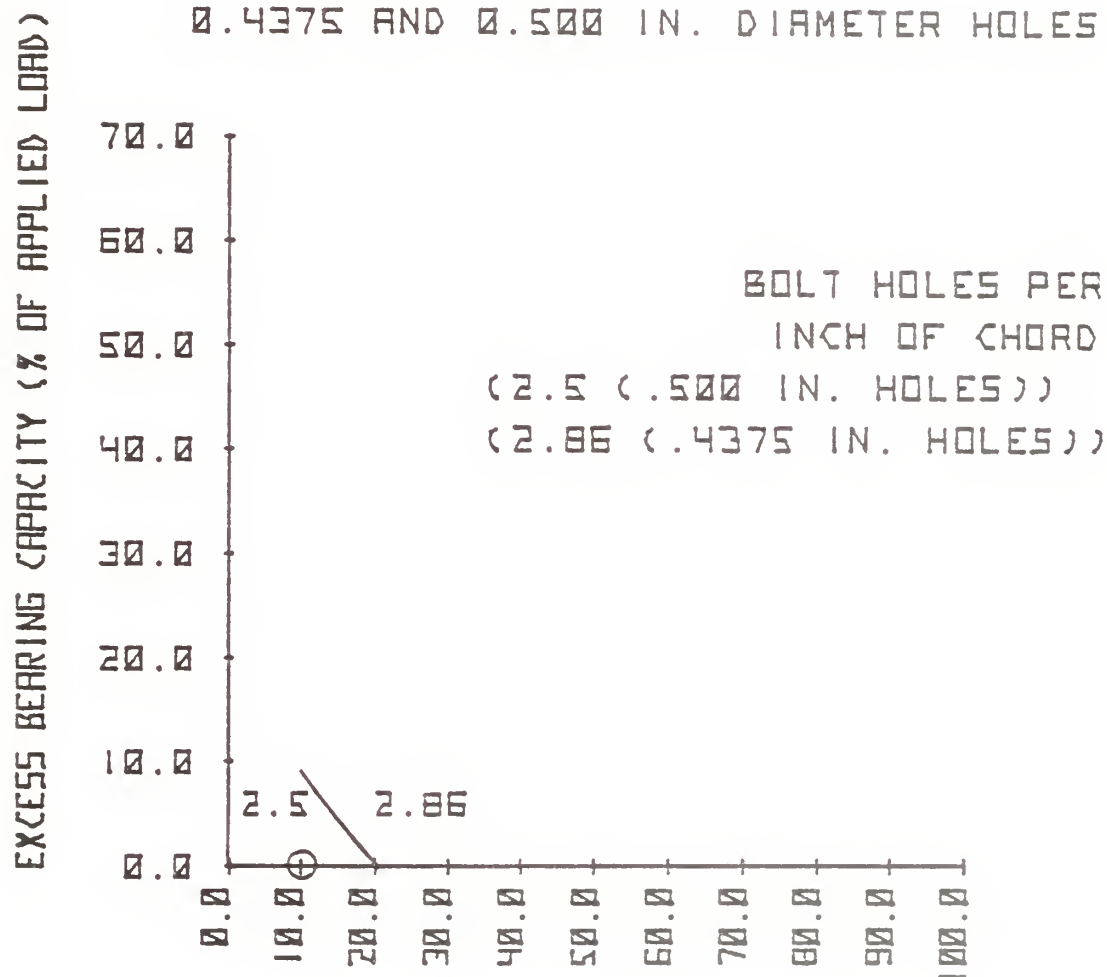


PER CENT ZERO DEG. PLYS AT OUTBOARD HOLE

FIGURE 32. VARIATION OF EXCESS BEARING CAPACITY  
WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP  
JOINTS WITH 0.375 IN. DIAMETER BOLT HOLES



VARIATION OF EXCESS BEARING CAPACITY  
WITH LAMINATE COMPOSITION  
NON-BUFFER STRIP JOINT  
0.4375 AND 0.500 IN. DIAMETER HOLES



PER CENT ZERO DEG. PLYS AT OUTBOARD HOLE

FIGURE 33. VARIATION OF EXCESS BEARING CAPACITY  
WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP  
JOINTS WITH 0.4375 AND 0.5 IN. DIAMETER BOLT HOLES

VARIATION OF JOINT WEIGHT WITH  
LAMINATE COMPOSITION  
0.25 IN. DIAMETER HOLES  
NON-BUFFER STRIP JOINT

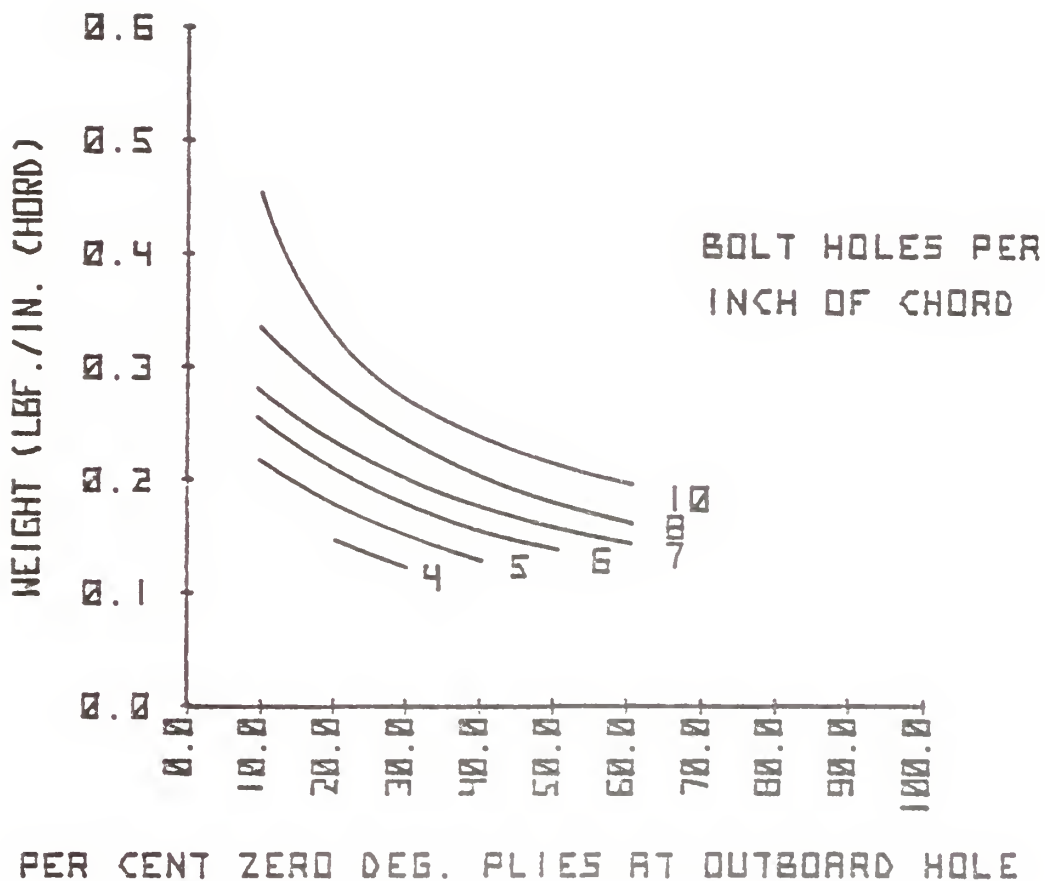


FIGURE 34. VARIATION OF JOINT WEIGHT WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP JOINTS WITH 0.25 IN. DIAMETER BOLT HOLES

VARIATION OF JOINT WEIGHT WITH  
LAMINATE COMPOSITION  
0.375 IN. DIAMETER HOLES  
NON-BUFFER STRIP JOINT

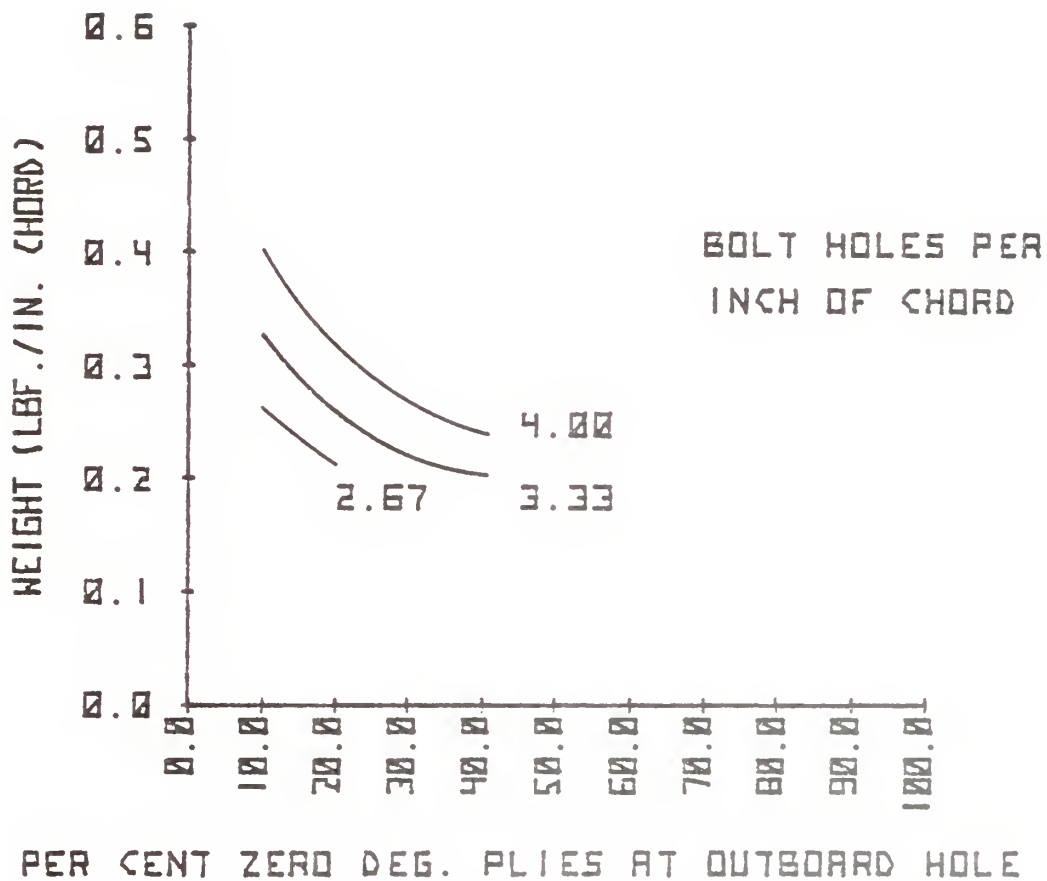


FIGURE 35. VARIATION OF JOINT WEIGHT WITH LAMINATE COMPOSITION FOR NON-BUFFER STRIP JOINTS WITH 0.375 IN. DIAMETER BOLT HOLES

VARIATION OF JOINT WEIGHT WITH  
LAMINATE COMPOSITION  
0.4375 AND 0.500 IN. DIAMETER HOLES  
NON-BUFFER STRIP JOINT

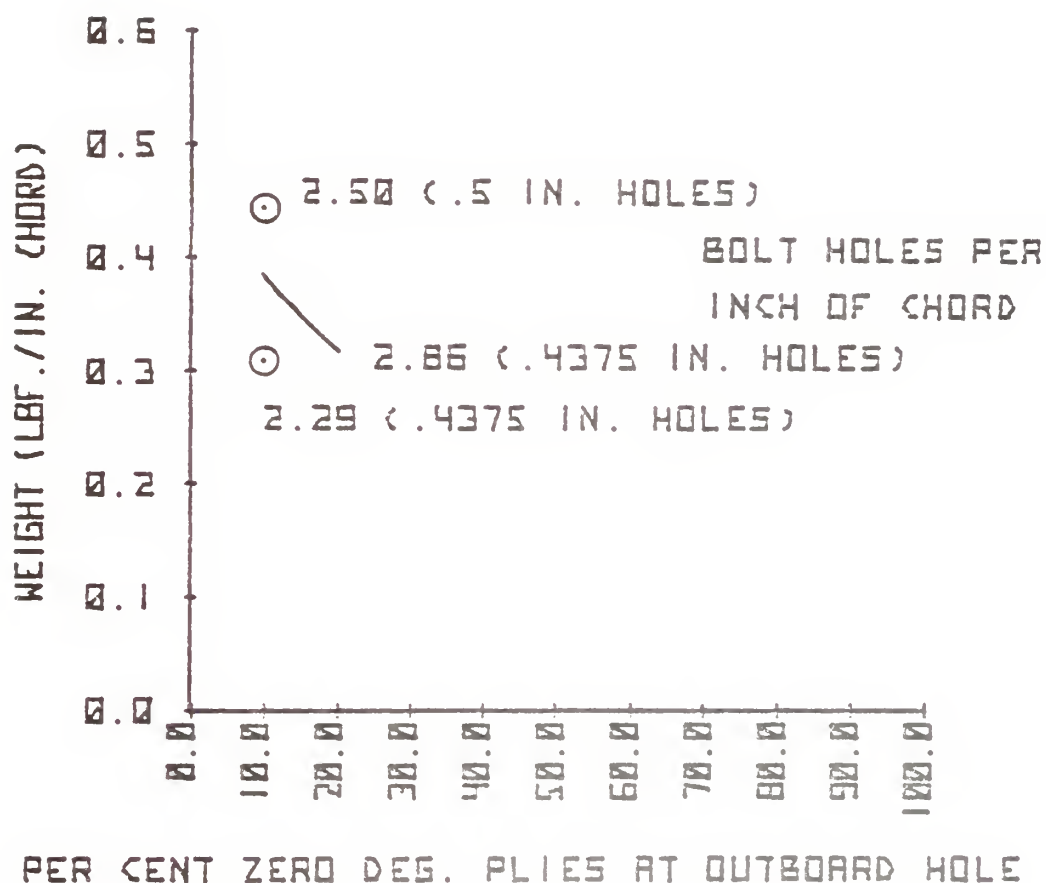
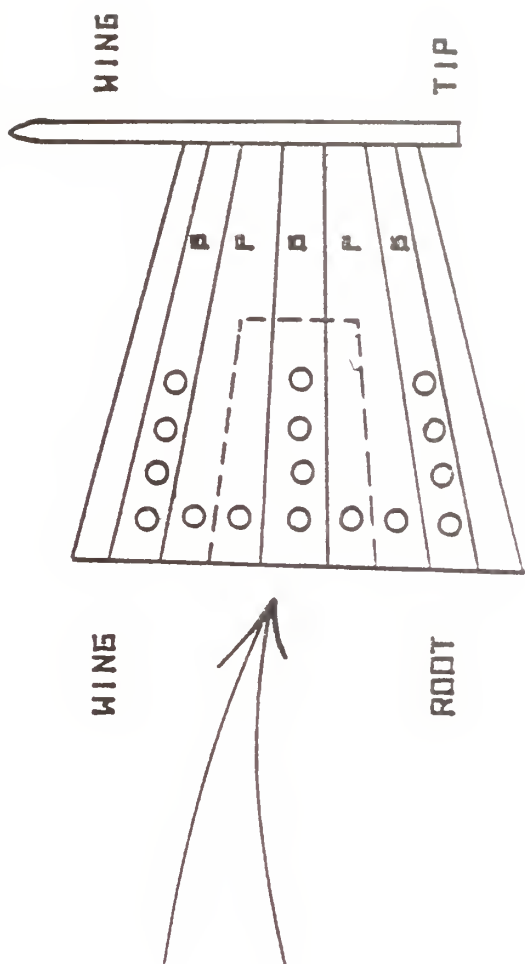
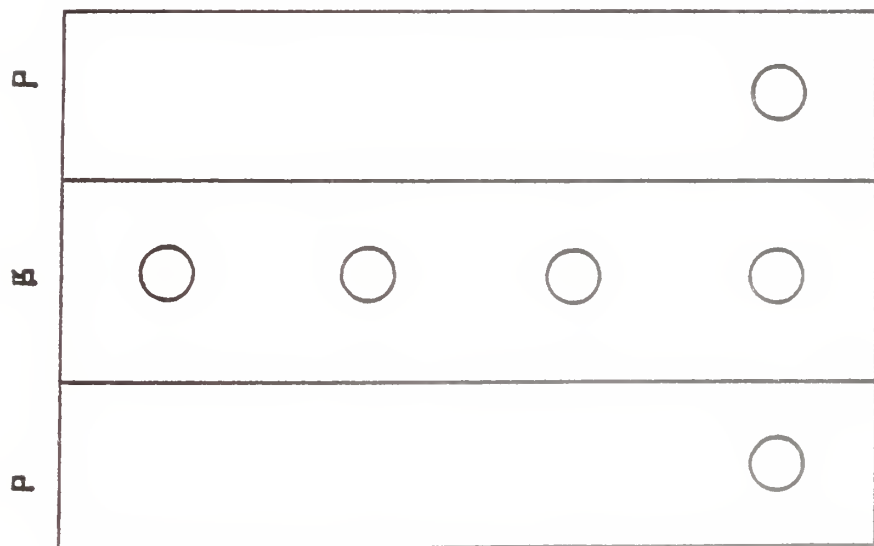


FIGURE 36. VARIATION OF JOINT WEIGHT WITH LAMINATE  
COMPOSITION FOR NON-BUFFER STRIP JOINTS WITH 0.4375  
AND 0.5 IN. DIAMETER BOLT HOLES

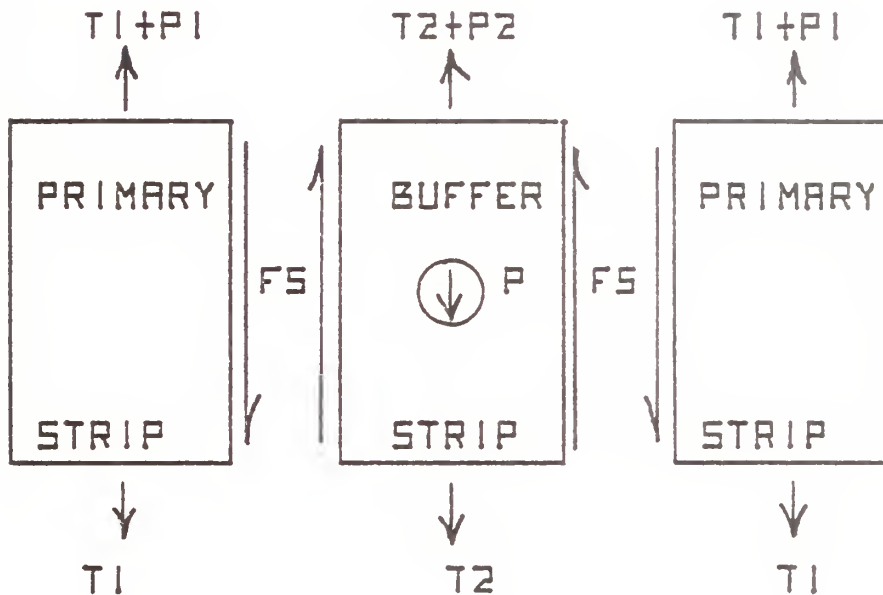


P = PRIMARY STRIP  
B = BUFFER STRIP

PER CENT ZERO DEG. PLIES IN PRIMARY STRIPS  
DECREASES FROM VALUE AT OUTBOARD BOLT TO  
ZERO AT INBOARD ROW OF BOLTS

FIGURE 37. SCHEMATIC OF A WING WITH A BUFFER STRIP JOINT

# MECHANISM BY WHICH BOLT LOADS ARE REACTED IN A BUFFER STRIP JOINT



$T_1$  = TENSILE LOAD IN PRIMARY STRIP

$T_2$  = TENSILE LOAD IN BUFFER STRIP

$P$  = BOLT LOAD APPLIED AT HOLE

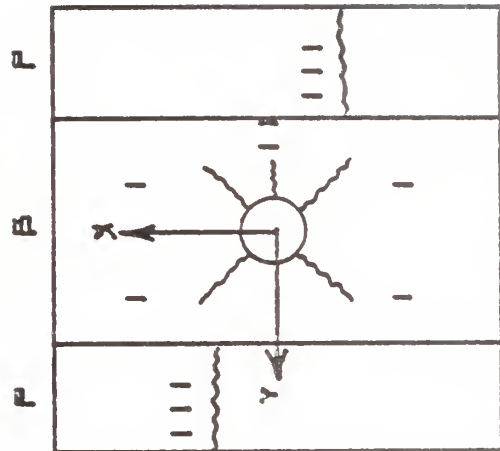
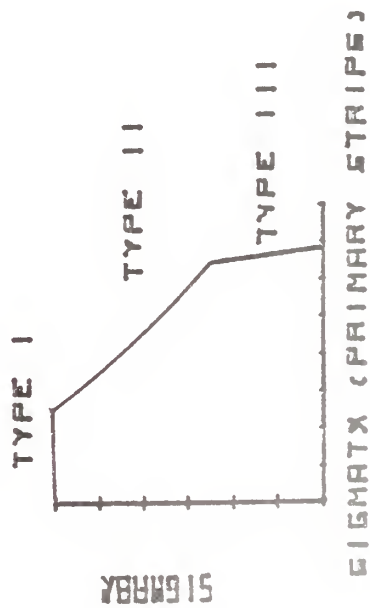
$P_1$  = BOLT LOAD REACTED IN PRIMARY STRIP

$P_2$  = BOLT LOAD REACTED IN BUFFER STRIP

$FS$  = SHEAR PASSING  $P_1$  TO PRIMARY STRIP

FIGURE 38. MECHANISM BY WHICH BOLT LOADS ARE  
REACTED IN A BUFFER STRIP JOINT

# BUFFER STRIP FAILURE MODES



TYPE I AT 45 DEG. TO X AXIS

B = BUFFER STRIP

EFFECT OF STRESS ON FAILURE TYPE P = PRIMARY STRIP

FAILURE TYPE DEFINITIONS

FIGURE 39. DESCRIPTION OF THE EXPECTED BUFFER STRIP FAILURE MODES

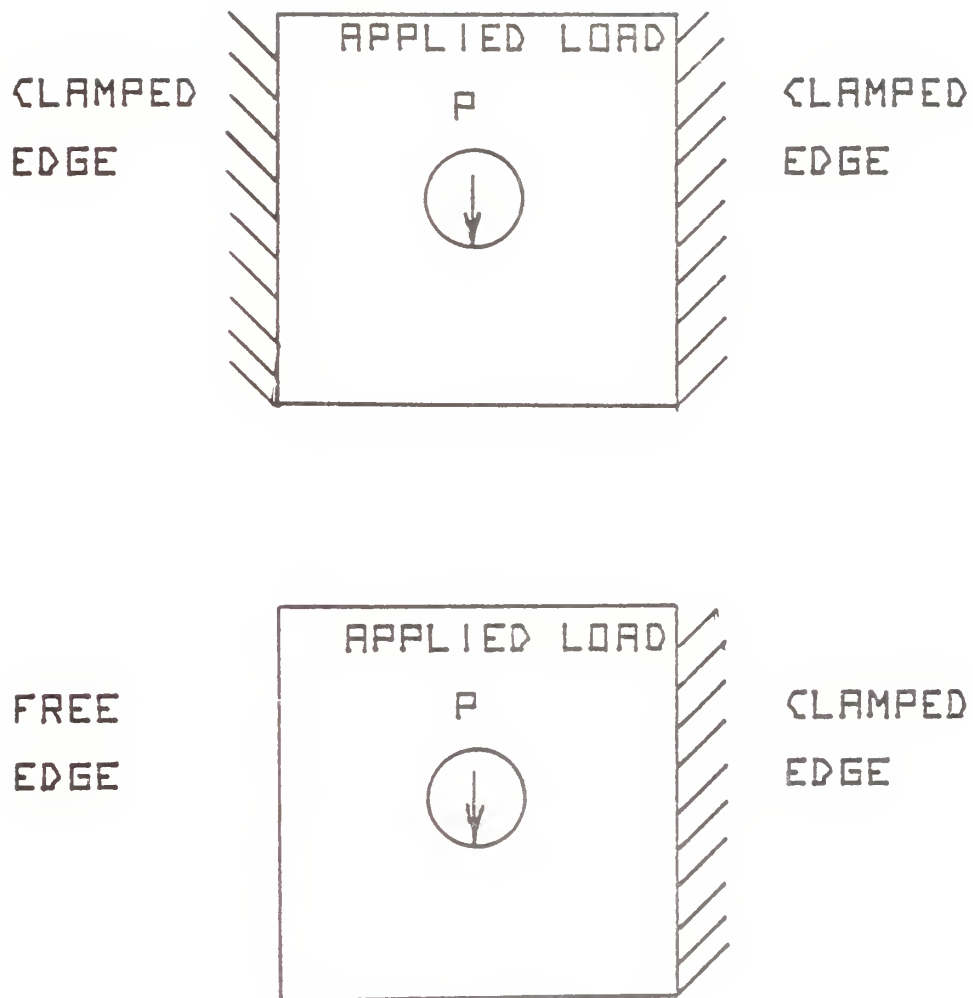


FIGURE 40. SCHEMATIC OF SHEAR LOADING TEST SPECIMENS



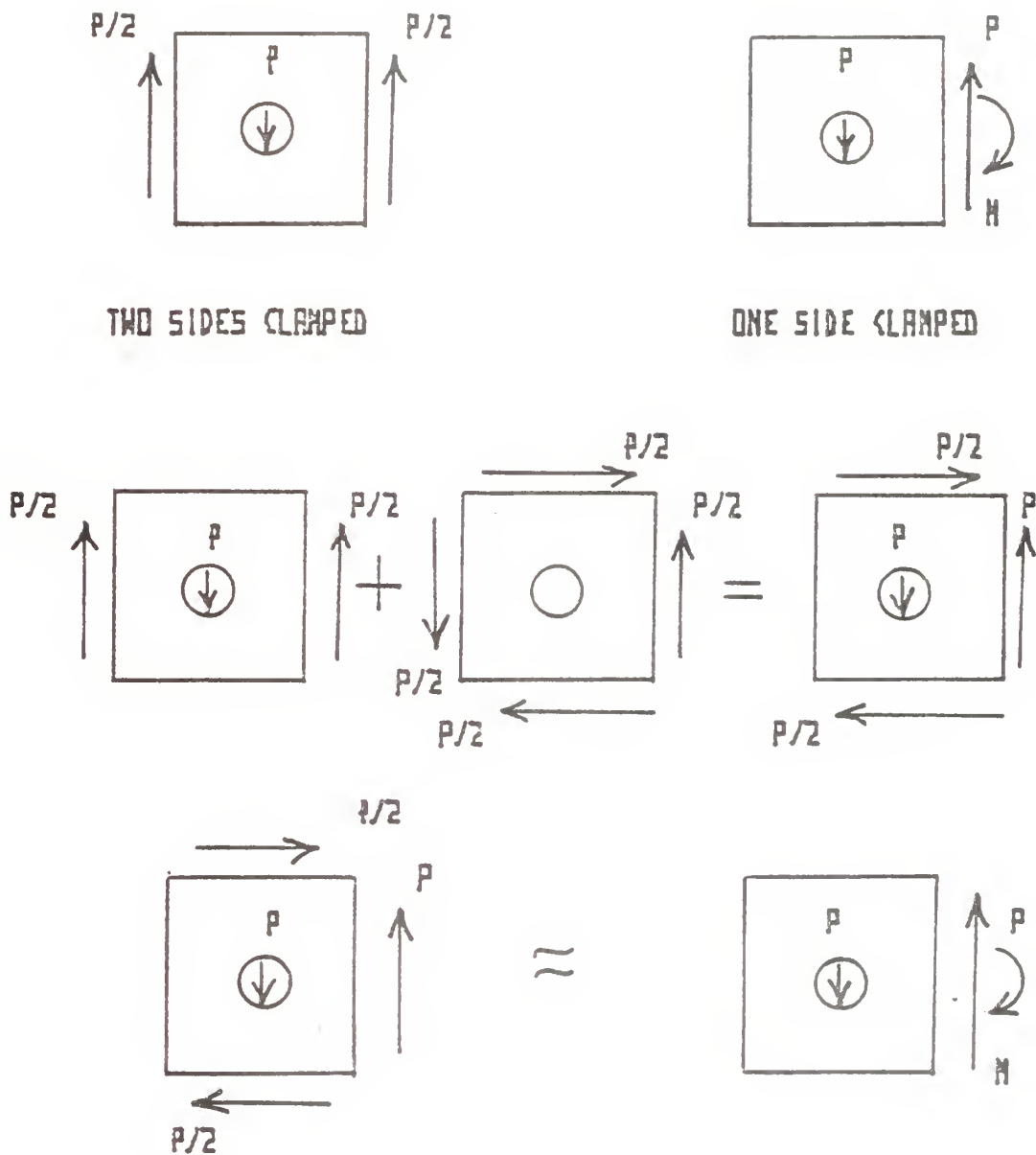


FIGURE 41. SCHEMATIC OF THE SUPERPOSITION USED TO  
DETERMINE SHEAR EFFECTS ON A BUFFER STRIP  
WITH A CENTRAL HOLE

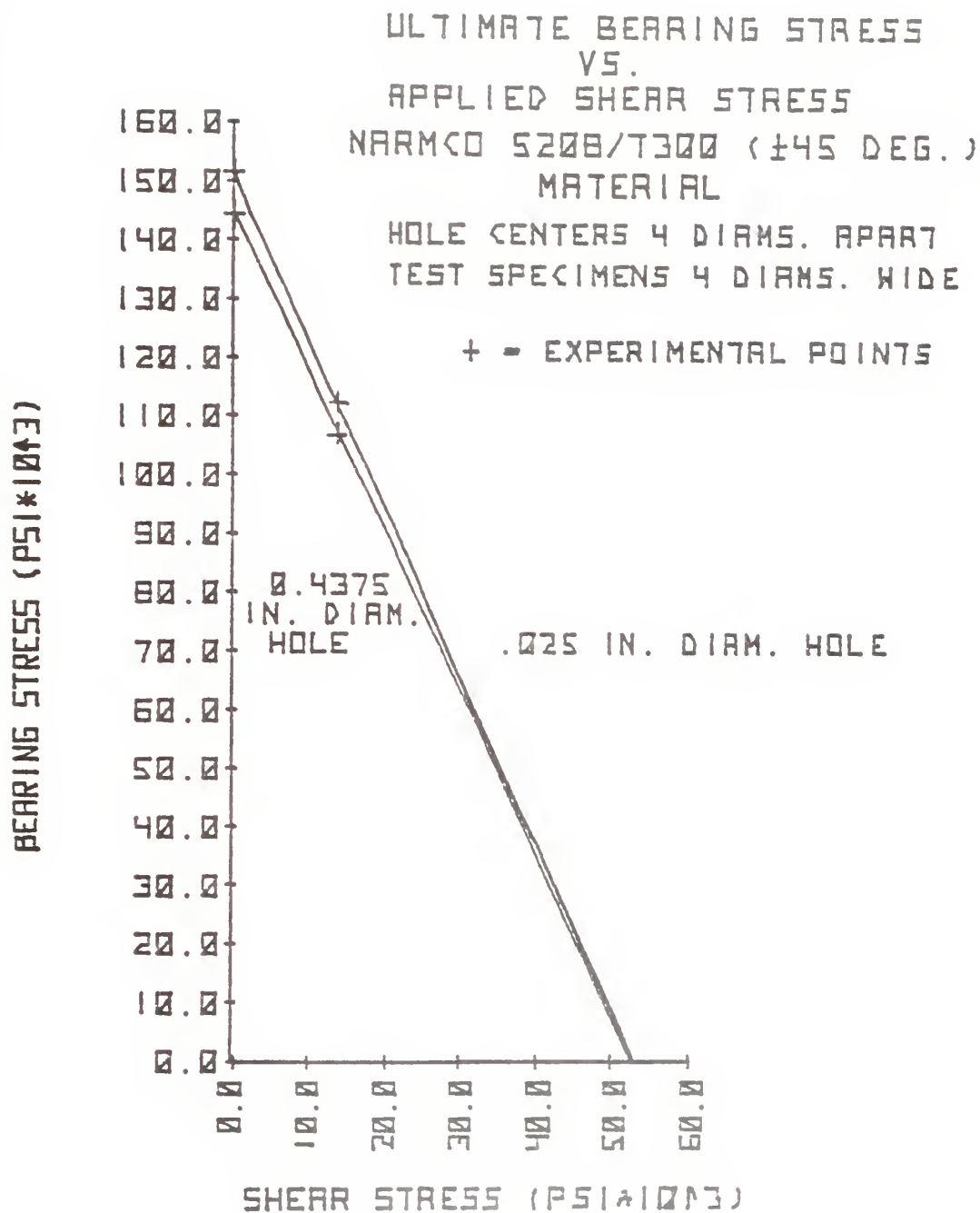


FIGURE 42. ULTIMATE BEARING STRESS-SHEAR STRESS  
 INTERACTION CURVE FOR A FOUR HOLE DIAMETER  
 SQUARE PLATE OF NARMCO 5203/T300 [ $\pm 45$ ]  
 MATERIAL WITH A CENTRAL HOLE

# VARIATION OF ULTIMATE BEARING STRESS WITH BYPASS STRESS IN THE PRIMARY STRIPS - BUFFER STRIP JOINT

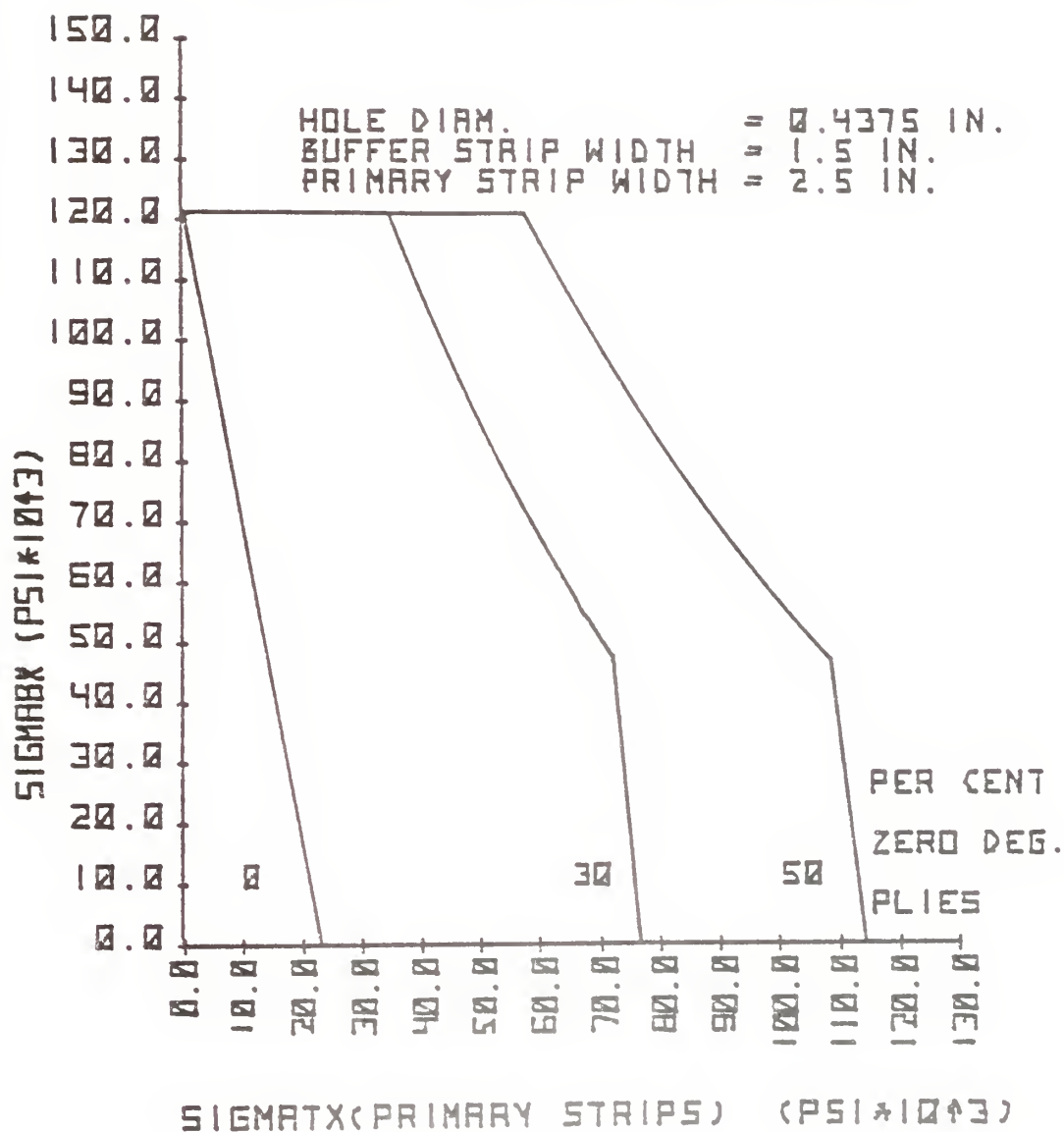


FIGURE 43. ULTIMATE STRESS INTERACTION CURVE FOR A BUFFER STRIP JOINT MADE FROM NARMCO 5208/T300  $[0/\pm 45]$  MATERIAL WITH 2.5 IN. WIDE PRIMARY STRIPS, A 1.5 IN. WIDE BUFFER STRIP, AND A 0.4375 IN. DIAMETER CENTRAL HOLE

VARIATION OF ULTIMATE BEARING STRESS  
WITH BYPASS STRESS IN THE PRIMARY  
STRIPS - BUFFER STRIP JOINT

HOLE DIAM. = 0.250 IN.  
BUFFER STRIP WIDTH = 4.00 DIAM.  
PRIMARY STRIP WIDTH = 3.33 DIAM.

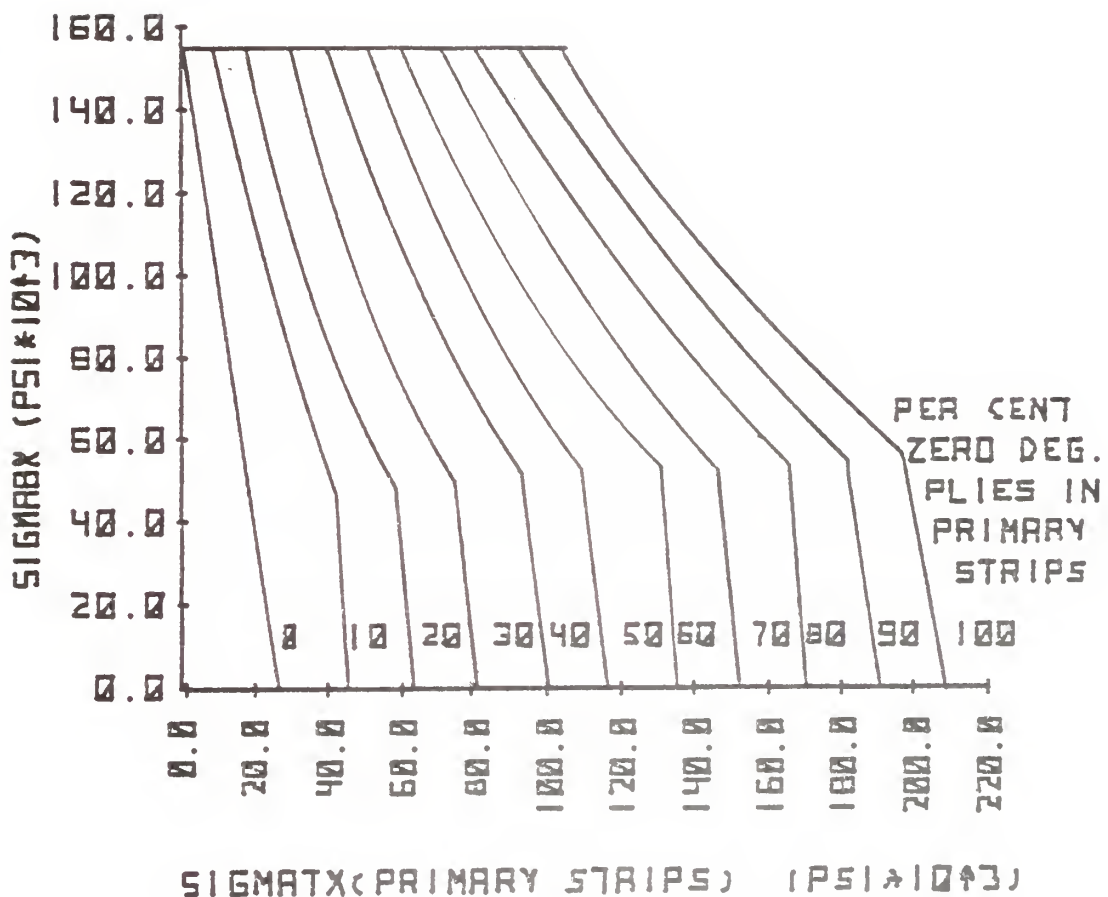


FIGURE 44. ULTIMATE STRESS INTERACTION CURVE FOR A  
1.0 IN. LONG BUFFER STRIP PLATE MADE FROM NARMCO  
5208/T300  $[0/\pm 45]$  MATERIAL WITH 0.833 IN. WIDE PRIMARY  
STRIPS, A 1.0 IN. WIDE BUFFER STRIP, AND A 0.25 IN.  
DIAMETER CENTRAL HOLE

# VARIATION OF ULTIMATE BEARING STRESS WITH BYPASS STRESS IN THE PRIMARY STRIPS - BUFFER STRIP JOINT

HOLE DIAM. = 0.4375 IN.  
BUFFER STRIP WIDTH = 4.00 DIAM.  
PRIMARY STRIP WIDTH = 3.33 DIAM.

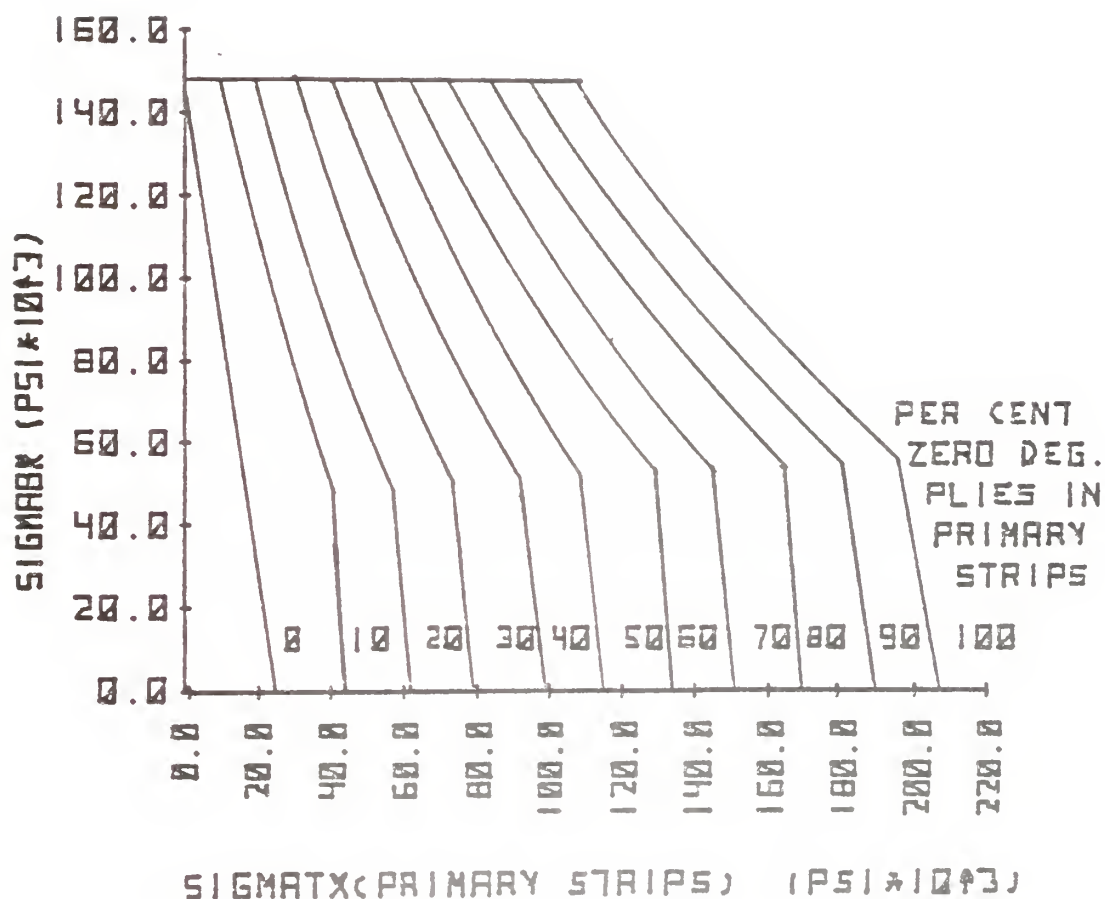
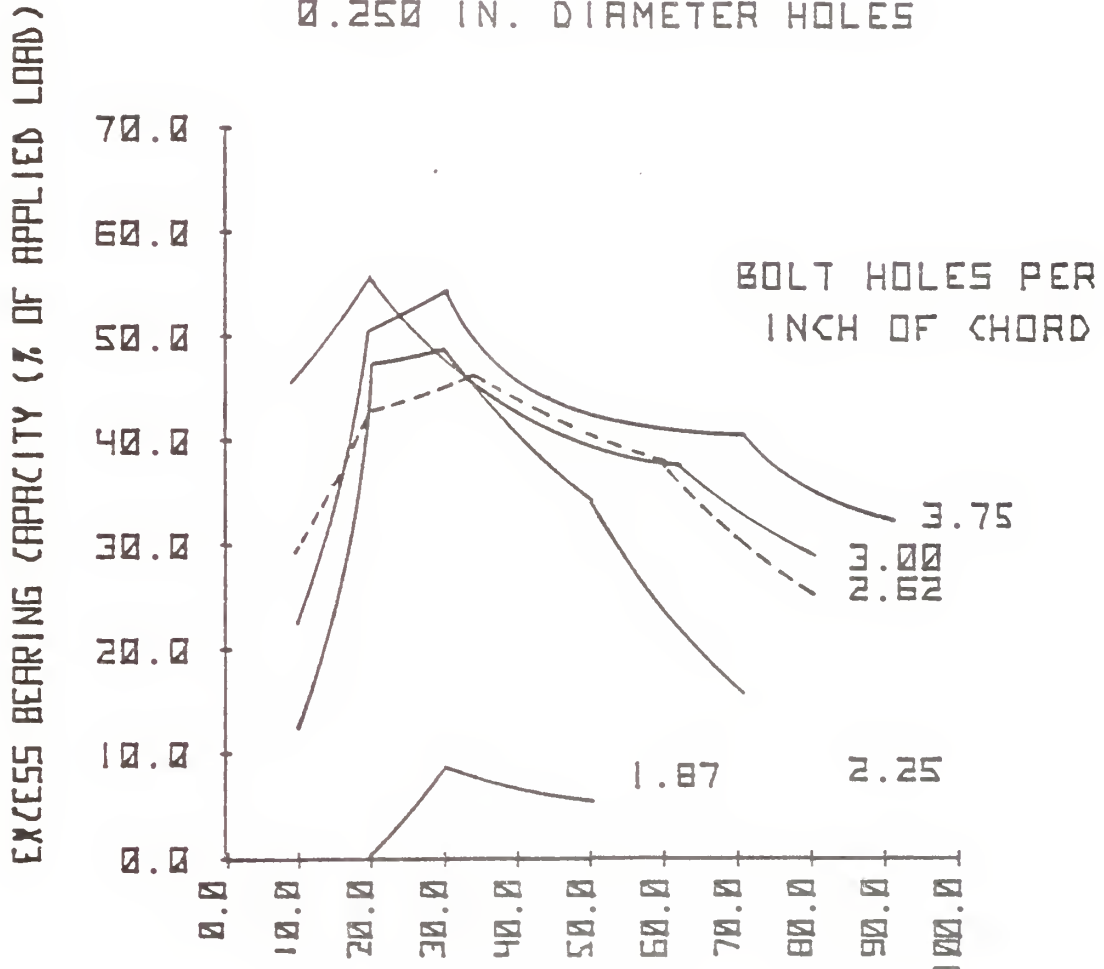


FIGURE 45. ULTIMATE STRESS INTERACTION CURVE FOR A  
1.75 IN. LONG BUFFER STRIP PLATE MADE FROM NARMCO  
5208/T300  $[0/\pm 45]$  MATERIAL WITH 1.46 IN. WIDE PRIMARY  
STRIPS, A 1.75 IN. WIDE BUFFER STRIP, AND A 0.4375  
IN. DIAMETER CENTRAL HOLE

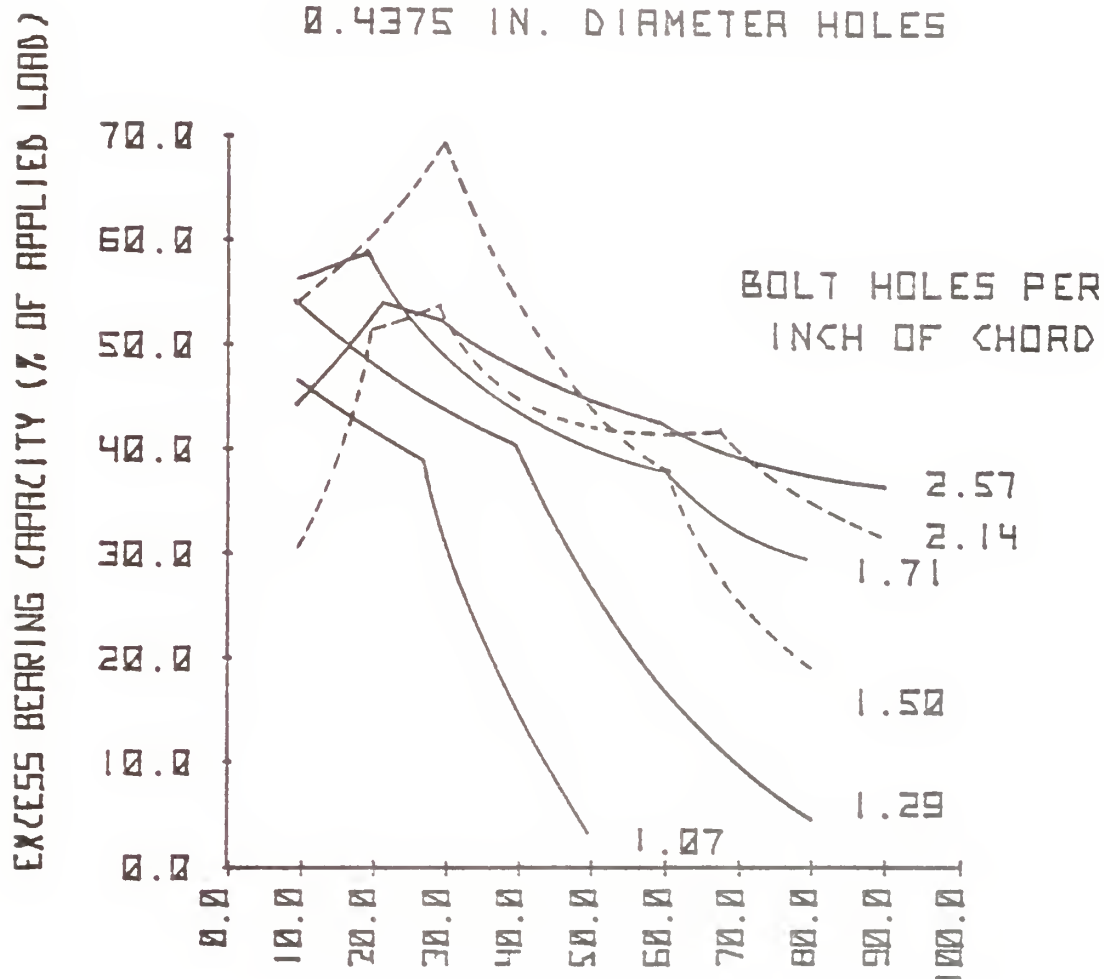
VARIATION OF EXCESS BEARING CAPACITY  
WITH LAMINATE COMPOSITION  
BUFFER STRIP JOINT  
0.250 IN. DIAMETER HOLES



PER CENT ZERO DEG. PLIES AT OUTBOARD HOLE

FIGURE 46. VARIATION OF EXCESS BEARING CAPACITY WITH LAMINATE COMPOSITION FOR BUFFER STRIP JOINTS WITH 0.25 IN. DIAMETER HOLES

VARIATION OF EXCESS BEARING CAPACITY  
WITH LAMINATE COMPOSITION  
BUFFER STRIP JOINT  
0.4375 IN. DIAMETER HOLES



PER CENT ZERO DEG. PLIES AT OUTBOARD HOLE

FIGURE 47. VARIATION OF EXCESS BEARING CAPACITY WITH LAMINATE COMPOSITION FOR BUFFER STRIP JOINTS WITH 0.4375 IN. DIAMETER HOLES

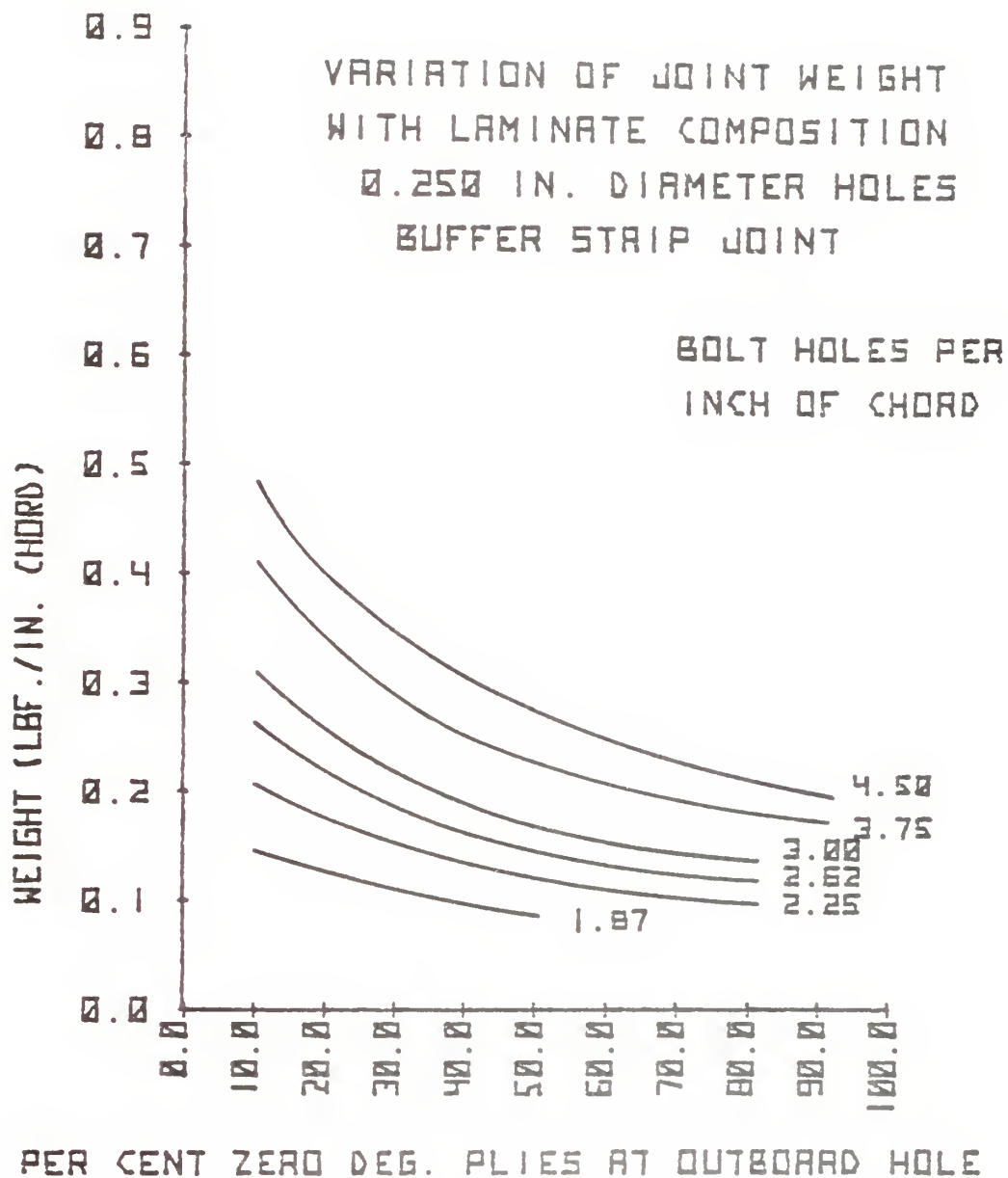


FIGURE 48. VARIATION OF JOINT WEIGHT WITH LAMINATE COMPOSITION FOR BUFFER STRIP JOINTS WITH 0.25 IN. DIAMETER HOLES



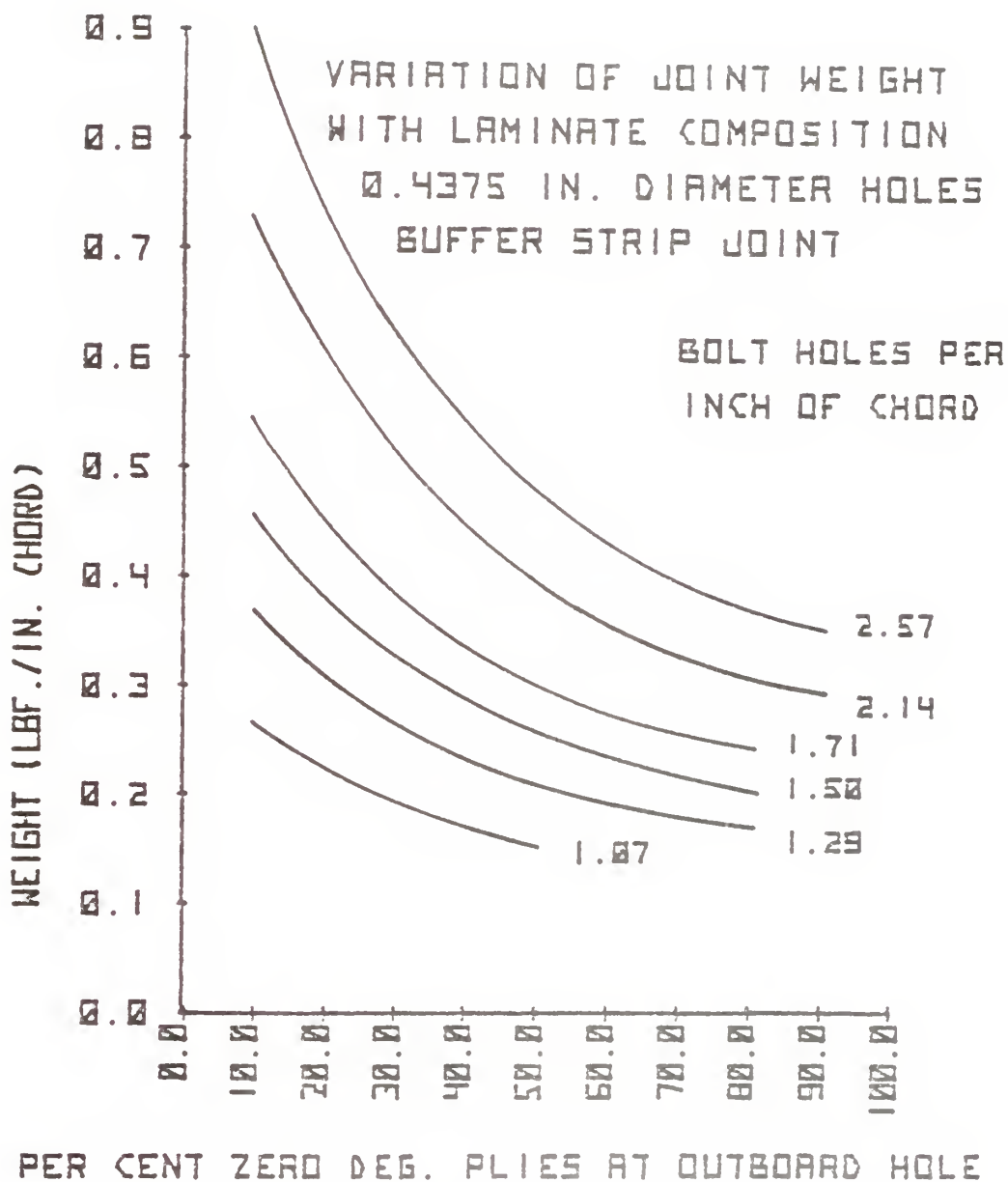


FIGURE 49. VARIATION OF JOINT WEIGHT WITH LAMINATE COMPOSITION FOR BUFFER STRIP JOINTS WITH 0.4375 IN. DIAMETER HOLES

TABLE I

## SUMMARY OF DESIGN CONDITIONS AND ASSUMPTIONS

1. The joints are made from NARMCO 5208/T300  $[0/\pm 45]$  graphite-epoxy laminated material.
2. Skin thickness varies linearly within a joint.
3. All bolt holes in a joint are of the same diameter.
4. The interbolt strain level is 3000 micro-in./in.
5. In the theoretical developments for both buffer strip and non-buffer strip joints it was assumed that only tensile and shear loads were to be carried.
6. Each row of bolts reacts an equal portion of the applied tensile load.
7. The applied shear load is reacted by the inboard row of bolts.
8. The minimum number of rows of bolts in any joint is three.
9. Wing taper is disregarded.
10. The inboard row of bolts is in all  $\pm 45$  degree laminate.
11. The maximum joint length is ten inches.
12. In the non-buffer strip joints, there is a four-hole-diameter spacing between adjacent rows of bolt hole centers.
13. In the non-buffer strip joint, the laminate between the inboard and next to inboard bolt holes must contain at least five per cent zero degree plies.
14. In the buffer strip joints, the buffer strip width is

four hole diameters. The primary strips are each 3.335 hole diameters wide.

15. Weight and excess bearing capacity calculations were made for assumed load conditions  $N_x = 20,000$  lbf./in. and  $N_{xy} = 0$ .

# APPENDIX A

```

PROGRAM ISANIS
** THIS PROGRAM ANALYZES THE STATE OF STRESS AND STRAIN IN A TWO
** DIMENSIONAL BODY COMPOSED OF EITHER ISOTROPIC OR
** ORTHOTROPIC MATERIALS. THE ANALYSIS IS PERFORMED BY THE FINITE
** ELEMENT METHOD USING ISOPARAMETRIC ELEMENTS.
** ISANIS IS A MODIFICATION OF PLISOP CODED BY PROFESSOR
** GILLES CANTIN AT THE NAVAL POSTGRADUATE SCHOOL IN 1972.
** ISANIS EXPANDS PLISOP TO DEAL WITH ORTHOTROPIC ELEMENTS.
** CODED BY JAMES M. GILL, APRIL 1975
*****
INPUT CARDS NEEDED
TITLE CARD (20A4) ONE CARD PER PROBLEM
PROBLEM PARAMETERS (10I5,5X,I5) ONE CARD PER PROBLEM
COLUMN VARIABLE
1-5 NEL NUMBER OF ELEMENTS. NEL MAX. IS 182
6-10 NJT NUMBER OF JOINTS. (MAX. IS 217)
11-15 NMAT NUMBER OF MATERIALS. (MAX IS 10)
16-20 NCLOAD NUMBER OF CONCENTRATED LOADS
21-25 NPBC NUMBER OF JOINTS WITH BOUNDARY CONDITIONS
26-30 NSTRES ISOTROPIC MATERIAL PROBLEM TYPE. NSTRES
= 0 MEANS ISOTROPIC PLANE STRESS.
= 1 MEANS ISOTROPIC PLANE STRAIN.
FOR PLANE STRAIN, THICKNESS
WILL BE SET EQUAL TO 1.0
NGP NUMBER OF GAUSS POINTS TO BE USED IN THE
INTEGRATION. (MAX. IS 5)
NTLD ISOTROPIC THERMAL LOAD INDICATOR. NTLD = 1
MEANS THERE IS AN ISOTROPIC THERMAL LOAD.
NTLD=0 MEANS NO ISOTROPIC THERMAL LOADS.
NTBY BODY FORCE LOAD INDICATOR. NTBY = 1 MEANS
A BODY FORCE LOAD IS PRESENT. NTBY=0
MEANS NO BODY FORCE LOADS ARE PRESENT.
NTIN INITIAL STRETCH INDICATOR. NTIN = 1 MEANS
INITIAL STRETCH IS TO BE CONSIDERED.
NTIN=0 MEANS NO INITIAL STRETCH.
IF (NTLD+NTBY+NTIN) IS NOT EQUAL TO ZERO, NJT CARDS WILL BE
NEEDED TO INPUT THE CONSISTENT LOAD VECTOR USED TO GENERATE
THESE CONDITIONS. PROGRAM ISLOAD CAN BE USED TO GENERATE
THIS VECTOR.

```

56-60	NLEL NUMBER OF LAMINATE TYPES WHOSE MATERIAL PROPERTIES ARE TO BE CALCULATED FROM INPUT LAMINA PROPERTIES. (MAX. 50)	MAIN 450
JOINT CARDS	(15,5X,4G15.8,I5)	MAIN 460
COLUMN	VARIABLE	MAIN 470
1-5	JOINT NUMBER	MAIN 480
11-25	X COORDINATE OF THE JOINT (CARTESIAN COORD.)	MAIN 490
26-40	RADIUS VECTOR OF THE JOINT (POLAR COORD.)	MAIN 500
41-55	Y-COORDINATE OF THE JOINT (CARTESIAN COORD.)	MAIN 510
56-70	POLAR ANGLE IN DEGREES (POLAR COORD.)	MAIN 520
70-75	X-COMPONENT OF THE LOAD AT THE JOINT (CART)	MAIN 530
	Y-COMPONENT OF THE LOAD AT THE JOINT (CART)	MAIN 540
	COORDINATE SYSTEM INDICATOR. IND. = ZERO IF MAIN	MAIN 550
	THE COORDINATE SYSTEM IS CARTESIAN, IF IND	MAIN 560
	IS NOT ZERO, THE COORDINATE SYSTEM IS POLAR	MAIN 570
		MAIN 580
		MAIN 590
		MAIN 600
		MAIN 610
		MAIN 620
		MAIN 630
		MAIN 640
		MAIN 650
		MAIN 660
		MAIN 670
		MAIN 680
		MAIN 690
		MAIN 700
		MAIN 710
		MAIN 720
		MAIN 730
		MAIN 740
		MAIN 750
		MAIN 760
		MAIN 770
		MAIN 780
		MAIN 790
		MAIN 800
		MAIN 810
		MAIN 820
		MAIN 830
		MAIN 840
		MAIN 850
		MAIN 860
		MAIN 870
		MAIN 880
		MAIN 890
		MAIN 900
		MAIN 910
		MAIN 920

MATERIAL PROPERTIES CARDS(2I5,7F10.2)	NMAT CARDS PER PROBLEM
VARIABLE	
MATERIAL	
INDK EQ.0 (ISOTROPIC), EQ.1 (ORTHOTROPIC)	
YOUNG'S MODULUS (ISOTROPIC MATERIAL)	
E(L) (ORTHOTROPIC MATERIAL)	
POISSON'S RATIO (ISOTROPIC MATERIAL)	
E(T) (ORTHOTROPIC MATERIAL)	
NOT USED (ISOTROPIC MATERIAL)	
NU(LT) (ORTHOTROPIC MATERIAL)	
NOT USED (ISOTROPIC MATERIAL)	
NU(TL) (ORTHOTROPIC MATERIAL)	
FOR ORTHOTROPIC MATERIALS, ONLY 3 OF THE ABOVE 4 ENGIN.	
CONSTANTS (E(L),E(T),NU(LT),NU(TL)) NEED BE KNOWN. FOR AN	
UNKNOWN CONSTANT ENTER -1.0 IN THE APPROPRIATE COLUMN.	
NOT USED (ISOTROPIC MATERIALS)	
G(LT) (ORTHOTROPIC MATERIALS)	
ALPHA COEFF. OF ISOTROPIC THERMAL EXPANSION	
NOT USED FOR THERMALLY ANISOTROPIC MATERIALS	
MATERIAL THICKNESS (USED FOR ELEMENTS	
WHOSE MATERIAL PROPERTIES ARE NOT TO BE	
CALCULATED FROM INPUT LAMINA PROPERTIES.)	

51-60	
61-70	
71-80	

ELEMENT CARDS	(15I4,F10.5,I5)	NEL CARDS PER PROBLEM
1-4	ELEMENT IDENTIFICATION NUMBER	
5-52	ELEMENT CONNECTIVITY (COUNTER-CLOCKWISE)	
53-56	ELEMENT KIND - USE	
	1 FOR A FOUR NODAL POINT ELEMENT	
	2 FOR AN EIGHT NODAL POINT ELEMENT	
	3 FOR A TWELVE NODAL POINT ELEMENT	





```

C      CONSISTENT LOAD VECTOR (I10,2G25.16)      NJT CARDS PER PROBLEM
C      NO CARDS NEEDED IF (NTLD+NTBY+NTIN)=0.
C      COL. VARIABLE
C      1-10 JOINT NUMBER
C      11-35 X COMPONENT OF LOAD
C      36-70 Y COMPONENT OF LOAD
C
C      AS MANY PROBLEMS AS ONE DESIRES MAY BE STOPPED, BUT THE LAST CARD OF
C      A RUN MUST CONTAIN THE WORD STOP IN THE FIRST FOUR COLUMNS
C
C      MAIN PROGRAM
C      IMPLICIT REAL*8(A-H,O-Z)
C      IMPLICIT INTEGER*2(I-N)
C      INTEGER *4LMK,LML,NBAND2
C      COMMON /SQL/ BGK(434,86),ALOAD(434),ABGN
C      COMMON /MDIM/ MEL,MJT,MMT,MBD,MLEL,MLYR
C      COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCN(182,15),NBC(217,2),NSTR
C      1ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL
C      1COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCCN(10,7),TITLE(10),ANGL
C      1(182),THNP(217)
C      COMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)
C      COMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)
C      DIMENSION REACT(218,2),NCODE(217)
C      EQUIVALENCE (BGK(1,1),REACT(1,1))
C      MEL = 182
C      MJT = 217
C      MMT = 10
C      MBD = 86
C      MLEL = 50
C      MLYR = 100
C      PI = 3.14159265359DU
C      ZRO = 0.0DU
C      1 NBAND = 0
C      NBAND2 = NBAND
C
C      DO 2 I=1,MJT
C      2 NCODE(I) = 0
C
C      CALL INPUT
C      NEQ = 2*NJT
C      NPEN = NCON(1,13)*4
C
C      DO 5 I=1,NEL
C      5 LM(J) = NCON(I,J)
C
C      DO 3 J=1,NPEN
C      3 LM(J) = NCON(I,J)

```

```

C      NPELM = NPEL-1
C
C      DC 4 K=1,NPELM
C      LMK = LM(K)
C      JK = K+1
C
C      DC 4 L=JK,NPEL
C      LML = LM(L)
C      4 NBAND2 = MAXO(NBAND2, IABS(LMK-LML))
C      5 CONTINUE
C
C      NBAND = NBAND2
C      NBAND = (NBAND+1)*2
C      IF (NBAND.GT.MBD) GO TO 16
C      WRITE (6,17) NBAND
C
C      DO 6 I=1,NEQ
C      ALOAD(I) = ZERO
C
C      DO 6 J=1,NBAND
C      BGK(I,J) = ZERO
C
C      CALL MEFGC (NPEL,NEQ)
C      CALL BCND (NBAND)
C      CALL LDLT (NEQ,NBAND)
C
C      DO 7 I=1,NPBC
C      IJT = NBC(I,1)
C      7 NCODE(IJT) = NBC(I,2)
C
C      DO 10 I=1,NJT
C      II = I*2-1
C      KODE = NCODE(I)
C      IF (KODE.EQ.7) GO TO 9
C      IF ((KODE.EQ.5).OR.(KODE.EQ.6)) GO TO 8
C      ALOAD(II) = CLOAD(I,1)+ALOAD(II)
C      8 II = II+1
C      IF ((KODE.EQ.4).OR.(KODE.EQ.6)) GO TO 10
C      ALOAD(II) = CLOAD(I,2)+ALOAD(II)
C      GO TO 10
C      9 ALF = CLOAD(I,2)*PI/180.0D0
C      CCSA = DCOS(ALF)
C      SINA = DSIN(ALF)
C      IIP = II+1
C      AI = ALOAD(II)*COSA+ALOAD(IIP)*SINA+CLOAD(I,1)

```



```

      A2 = ALOAD(IIP)*COSA-ALOAD(II)*SINA
      ALOAD(II) = A1
      ALCAD(IIP) = A2
10  CONTINUE
C
      SXL = ZRO
      SYL = ZRO
C
      DO 12 I=1,NJT
      IX = 2*I-1
      IY = IX+1
      IF (NCODE(I).EQ.7) GO TO 11
      SXL = SXL+ALOAD(IX)
      SYL = SYL+ALOAD(IY)
      GO TO 12
11  ALF = CLOAD(I,2)*PI/180.000
      COSA = DCOS(ALF)
      SINA = DSIN(ALF)
      SXL = SXL+ALOAD(IX)*COSA-ALOAD(IY)*SINA
      SYL = SYL+ALOAD(IX)*SINA+ALOAD(IY)*COSA
12  CONTINUE
C
      WRITE (6,24)
      WRITE (6,22) SXL,SYL
      CALL SOLV (NEQ,NBAND)
C
      DO 13 I=1,NPBC
      DC 13 J=1,2
13  REACT(I,J) = ZRO
C
      CALL DISPL
      WRITE (6,19)
C
      DC 14 I=1,NJT
      II = I*2-1
      III = II+1
14  WRITE (6,18) I,ALOAD(II),ALOAD(III)
C
      WRITE (6,20)
C
      DC 15 I=1,NPBC
15  WRITE (6,18) NBC(I,1),REACT(I,1),REACT(I,2)
C
      WRITE (6,21)
      MJP = MJT+1
      WRITE (6,22) REACT(MJP,1),REACT(MJP,2)
      CALL STRESS (NPBL)

```

MA IN2370  
 MA IN2380  
 MA IN2390  
 MA IN2400  
 MA IN2410  
 MA IN2420  
 MA IN2430  
 MA IN2440  
 MA IN2450  
 MA IN2460  
 MA IN2470  
 MA IN2480  
 MA IN2490  
 MA IN2500  
 MA IN2510  
 MA IN2520  
 MA IN2530  
 MA IN2540  
 MA IN2550  
 MA IN2560  
 MA IN2570  
 MA IN2580  
 MA IN2590  
 MA IN2600  
 MA IN2610  
 MA IN2620  
 MA IN2630  
 MA IN2640  
 MA IN2650  
 MA IN2660  
 MA IN2670  
 MA IN2680  
 MA IN2690  
 MA IN2700  
 MA IN2710  
 MA IN2720  
 MA IN2730  
 MA IN2740  
 MA IN2750  
 MA IN2760  
 MA IN2770  
 MA IN2780  
 MA IN2790  
 MA IN2800  
 MA IN2810  
 MA IN2820  
 MA IN2830  
 MA IN2840

```

C
16 GO TO 1
16 WRITE (6,25) MBD,NBAND
16 STOP

17 FORMAT (1X,'PROBLEM BANDWIDTH EQUALS ',I5)
18 FORMAT (2X,I3,2G15.4)
19 FORMAT (///,'DISPLACEMENTS',//,3X,' JOINT',1X,' U',12X,' V',//)
20 FORMAT (///,' REACTIONS',//,3X,' JOINT',1X,' RX',12X,' RY',//)
21 FCORMAT (//,' SUM OF THE REACTIONS AT THE SUPPORTS',//)
22 FORMAT (5X,2G15.4)
23 FCORMAT (5X,' MAX. BAND (' ,I13,' ) EXCEEDED; NBAND = ',I14)
24 FCORMAT (//,' SUM OF THE EXTERNAL LOADS',//)
24 END

C
SUBROUTINE INPUT
THIS SUBROUTINE READS INPUT DATA FOR THE PROBLEM
IMPLICIT REAL*8(A-H,O-Z)
IMPLICIT INTEGER*2(I-N)
COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTR
1 LES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL
COMMON /MDIM/ MEL,MJT,MMT,MBD,MLEL,MLYR
COMMON /FLPL/ COARD(217,2),CLQAD(217,2),ELCON(10,7),TITLE(10),ANGL
1(182),THNP(217)
COMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)
COMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)
DATA CHK/,STOP
ZFC = 0.0D0
PI = 3.14159265359D0
READ (5,30) TITLE
WRITE (6,31) TITLE
IF (TITLE(1).EQ.CHK) STOP

C
DO 1 I=1,MJT
THNP(I) = ZRO

C
DO 1 J=1,2
NBC(I,J) = 0
COARD(I,J) = ZRO
1 CLCAD(I,J) = ZRO

C
DC 2 I=1,MMT
INDK(I) = 0

C
DC 2 J=1,7
2 ELCON(I,J) = ZRO

C
DO 3 I=1,MEL

```

```

MAIN2850
MAIN2860
MAIN2870
MAIN2880
MAIN2890
MAIN2900
MAIN2910
MAIN2920
MAIN2930
MAIN2940
MAIN2950
MAIN2960
MAIN2970
INPT 10
INPT 20
INPT 30
INPT 40
INPT 50
INPT 60
INPT 70
INPT 80
INPT 90
INPT 100
INPT 110
INPT 120
INPT 130
INPT 140
INPT 150
INPT 160
INPT 170
INPT 180
INPT 190
INPT 200
INPT 210
INPT 220
INPT 230
INPT 240
INPT 250
INPT 260
INPT 270
INPT 280
INPT 290
INPT 300
INPT 310
INPT 320
INPT 330
INPT 340
INPT 350

```

```

C      ANGL(I) = ZRO
C      DO 3 J=1,15
C      3 NCON(I,J) = 0
C
C      DC 4 I=1,MLEL
C      LYRNO(I) = 0
C
C      DC 4 J=1,MLYR
C      LAMAT(I,J) = 0
C      ORLAM(I,J) = ZRO
C      4 TKLAM(I,J) = ZRO
C
C      READ PRCBLE PARAMETERS
C      READ (5,32) NEL,NJT,NMAT,NCLOAD,NPBC,NSTRES,NGP,NTLD,NTBY,NTIN,NLE
C      1L IF (NEL.GT.MEL) GO TO 23
C      IF (NJT.GT.MJT) GO TO 24
C      IF (NMAT.GT.MMT) GO TO 25
C      IF (NCLOAD.GT.MJT) GO TO 26
C      IF (NPBC.GT.MJT) GO TO 27
C      IF (NLEL.GT.MLEL) GO TO 28
C      WRITE (6,33) NEL,NJT,NMAT,NCLOAD,NPBC,NSTRES,NGP,NLEL
C
C      READ COORDINATES OF JOINTS AND CONCENTRATED LOADS
C      WRITE (6,34)
C
C      DC 5 I=1,NJT
C      READ (5,35) IJT,COORD(IJT,1),COORD(IJT,2),CLOAD(IJT,1),CLOAD(IJT,2
C      1),IND
C      IF (IND.EQ.0) GO TO 5
C      ALF = COORD(IJT,2)*PI/180.0D0
C      CGSA = DCOS(ALF)
C      SINA = DSIN(ALF)
C      XC = COORD(IJT,1)*COSA
C      YC = COORD(IJT,1)*SINA
C      CCARD(IJT,1) = XC
C      CCARD(IJT,2) = YC
C      5 CCNTINUE
C
C      DO 6 I=1,NJT
C      6 WRITE (6,36) I,COORD(I,1),COORD(I,2),CLOAD(I,1),CLOAD(I,2)
C

```

```

C      READ MATERIAL PROPERTIES CARDS
C
C      DO 7 I=1,NMAT
C      7 READ (5,37) IMAT,INDK(IMAT),(ELCON(IMAT,J),J=1,7)
C      WRITE (6,38)
C
C      DO 8 I=1,NMAT
C      8 WRITE (6,39) I,(ELCON(I,J),J=1,7),INDK(I)
C      CONTINUE
C
C      READ THE ELEMENT CARDS
C      WRITE (6,40)
C      WRITE (6,41)
C
C      DO 9 I=1,NEL
C      9 READ (5,42) IJT,(NCON(IJT,J),J=1,14),ANGL(IJT),NCON(IJT,15)
C
C      DO 10 I=1,NEL
C      10 WRITE (6,43) I,(NCON(I,J),J=1,14),ANGL(I),NCON(I,15)
C
C      READ BOUNDARY CONDITION CARDS
C      WRITE (6,44)
C
C      DO 11 I=1,NPBC
C      11 READ (5,45) (NBC(I,J),J=1,2)
C      WRITE (6,46) (NBC(I,J),J=1,2)
C
C      DO 12 I=1,NPBC
C      12 IF (NBC(I,1).LE.NBC(J,1)) GO TO 12
C      NT1 = NBC(I,1)
C      NT2 = NBC(I,2)
C      NBC(I,1) = NBC(J,1)
C      NBC(I,2) = NBC(J,2)
C      NBC(J,1) = NT1
C      NBC(J,2) = NT2
C      CONTINUE
C

```

```

C
NSV = 0
DO 13 I=1,NPBC
IF (NBC(I,2).NE.7) GO TO 13
NSV = NSV+1
13 CCNTINUE
C
NPBCM = NPBC-1
IF (NSV.EQ.0) GO TO 18
C
DO 16 I=1,NSV
C
DC 15 J=1,NPBCM
IF (NBC(J,2).NE.7) GO TO 15
NT1 = NBC(J,1)
C
DO 14 K=J,NPBCM
KP = K+1
NBC(K,1) = NBC(KP,1)
14 NBC(K,2) = NBC(KP,2)
C
NBC(NPBC,1) = NT1
NBC(NPBC,2) = 7
GO TO 16
15 CCNTINUE
C
16 CONTINUE
C
WRITE (6,44)
C
DC 17 I=1,NPBC
17 WRITE (6,46) (NBC(I,J),J=1,2)
C
18 CCNTINUE
IF (NLEL.EQ.0) GO TO 21
C
READ LAMINATE CARD DECK
C
C
DO 19 I=1,NLEL
READ (5,47) LAMTP,LYRNO(LAMTP)
L = LYRNO(LAMTP)
IF (L.GT.MLYR) GO TO 29
19 READ (5,48) ((LAMAT(LAMTP,J),TKLAM(LAMTP,J),ORLAM(LAMTP,J)),J=1,L)
CONTINUE
C
WRITE (6,49)
C

```

```

INPT1320
INPT1330
INPT1340
INPT1350
INPT1360
INPT1370
INPT1380
INPT1390
INPT1400
INPT1410
INPT1420
INPT1430
INPT1440
INPT1450
INPT1460
INPT1470
INPT1480
INPT1490
INPT1500
INPT1510
INPT1520
INPT1530
INPT1540
INPT1550
INPT1560
INPT1570
INPT1580
INPT1590
INPT1600
INPT1610
INPT1620
INPT1630
INPT1640
INPT1650
INPT1660
INPT1670
INPT1680
INPT1690
INPT1700
INPT1710
INPT1720
INPT1730
INPT1740
INPT1750
INPT1760
INPT1770
INPT1780
INPT1790

```

```

C
DC 20 I=1,NLEL
WRITE (6,50) I,LYRNO(I)
L = LYRNO(I)
C
DO 20 J=1,L
20 WRITE (6,51) J,LAMAT(I,J),TKLAM(I,J),ORLAM(I,J)
C
READ THE CONSISTENT LOAD VECTOR (IF REQUIRED)
C
21 IF ((NTLD+NTBY+NTIN).EQ.0) RETURN
C
DO 22 I=1,NJT
READ (5,52) IJT,CLX,CLY,THNP(I)
CLOAD(IJT,1) = CLOAD(IJT,1)+CLX
CLOAD(IJT,2) = CLOAD(IJT,2)+CLY
22 CCNTINUE
C
RETURN
23 WRITE (6,53) MEL,NEL
STOP
24 WRITE (6,54) MJT,NJT
STOP
25 WRITE (6,55) MMT,NMAT
STOP
26 WRITE (6,56) MJT,NCLOAD
STOP
27 WRITE (6,57) MJT,NPBC
STOP
28 WRITE (6,58) MLEL,NLEL
STOP
29 WRITE (6,59) MLYR,L
STOP
C
30 FCRMAT (10A8)
31 FCRMAT (1X,10A8)
32 FCRMAT (10I5,5X,15)
33 FCRMAT (//, , NUMBER OF ELEMENTS=,15,/, , TCTAL NUMBER OF JOINTS=,
1,15,/, , NUMBER OF MATERIALS=,15,/, , NUMBER OF CONCENTRATED LOAD
2S=,15,/, , NUMBER OF JOINTS WITH BOUNDARY CONDITIONS=,15,/, , TH
3E VALUE OF LAMINATE TYPES USED,15)
4 FCRMAT (//, , JOINT NUMBER,5X, , X COORDINATE,5X, , Y COORDINATE,7X
1, , X LOAD,9X, , Y LOAD,/)
35 FCRMAT (15,5X,4G15.8,15)
36 FCRMAT (5X,13,10X,G14.5,3X,G14.5,3X,G14.5)
37 FCRMAT (2I5,7F10.2)
38 FCRMAT (//, , MATERIAL PROPERTIES,/,/,1X, , MATERIAL
,2X, , F,E(L)
INPT1800
INPT1810
INPT1820
INPT1830
INPT1840
INPT1850
INPT1860
INPT1870
INPT1880
INPT1890
INPT1900
INPT1910
INPT1920
INPT1930
INPT1940
INPT1950
INPT1960
INPT1970
INPT1980
INPT1990
INPT2000
INPT2010
INPT2020
INPT2030
INPT2040
INPT2050
INPT2060
INPT2070
INPT2080
INPT2090
INPT2100
INPT2110
INPT2120
INPT2130
INPT2140
INPT2150
INPT2160
INPT2170
INPT2180
INPT2190
INPT2200
INPT2210
INPT2220
INPT2230
INPT2240
INPT2250
INPT2260
INPT2270

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1, 7X, 'NU, E(LT)', 5X, 'NU(TL)', 8X, 'NU(TL)', 10X, 'G(LT)', 12X, 'ALPHA', 9X INPT 2280
2, THICKNESS, 9X, 'INDK', INPT 2290
35 FCRMAT (, 1X, 13, 5X, 7(2X, G13.6), 2X, I5) INPT 2300
40 FCRMAT (, 1X, 13, 5X, 7(2X, G13.6), 2X, I5) INPT 2310
41 FCRMAT (, 1X, 13, 5X, 7(2X, G13.6), 2X, I5) INPT 2320
42 FCRMAT (, 1X, 13, 5X, 7(2X, G13.6), 2X, I5) INPT 2330
43 FCRMAT (, 1X, 13, 5X, 7(2X, G13.6), 2X, I5) INPT 2340
44 FCRMAT (, 1X, 13, 5X, 7(2X, G13.6), 2X, I5) INPT 2350
45 FCRMAT (, 1X, 13, 5X, 7(2X, G13.6), 2X, I5) INPT 2360
46 FCRMAT (, 1X, 13, 5X, 7(2X, G13.6), 2X, I5) INPT 2370
47 FCRMAT (, 1X, 13, 5X, 7(2X, G13.6), 2X, I5) INPT 2380
48 FCRMAT (, 1X, 13, 5X, 7(2X, G13.6), 2X, I5) INPT 2390
49 FCRMAT (, 1X, 13, 5X, 7(2X, G13.6), 2X, I5) INPT 2400
1 LAYERS, 1X, 'LAMINATE PROPERTIES', 1X, 'LAMINATE TYPE', 5X, 'NO OF INPT 2410
2 ORIENTATION, 1X, 'LAYER NUMBER', 5X, 'MATERIAL NO.', 5X, 'THICKNESS', 5X, INPT 2420
30 FCRMAT (4X, I5, 11X, I5) OF ELEMENTS EXCEEDED, NEL=, 114 INPT 2430
51 FCRMAT (30X, I5, 8X, I7, 10X, F9.3, 5X, F9.3) INPT 2440
52 FCRMAT (5X, I5, 2G25.16, 1G15.8) INPT 2450
53 FCRMAT (5X, I5, 2G25.16, 1G15.8) INPT 2460
1) FCRMAT (5X, I5, 2G25.16, 1G15.8) OF JOINTS EXCEEDED, NJT=, 114 INPT 2470
54 FCRMAT (5X, I5, 2G25.16, 1G15.8) OF JOINTS EXCEEDED, NJT=, 114 INPT 2480
55 FCRMAT (5X, I5, 2G25.16, 1G15.8) OF MATERIAL EXCEEDED, NMAT=, 114 INPT 2490
1) FCRMAT (5X, I5, 2G25.16, 1G15.8) OF C. LOADS EXC., NLOAD=, 114 INPT 2500
56 FCRMAT (5X, I5, 2G25.16, 1G15.8) OF JOINTS WITH BOUND. COND. EXCEEDED, INPT 2510
57 FCRMAT (5X, I5, 2G25.16, 1G15.8) OF JOINTS WITH BOUND. COND. EXCEEDED, INPT 2520
1 NPBC=, 114 INPT 2530
58 FCRMAT (5X, I5, 2G25.16, 1G15.8) OF LAMINATE TYPES EXCEEDED, NLEL=, INPT 2540
1 I5 INPT 2550
59 FCRMAT (5X, I5, 2G25.16, 1G15.8) OF LAMINATE LAYERS EXCEEDED, LYNRO = INPT 2560
1, I5 INPT 2570
ENC INPT 2580
SUBROUTINE MERGE (NPEL, NEQ) INPT 2590
IMPLICIT REAL*8(A-H, O-Z) MERG 10
IMPLICIT INTEGER*2(I-N) MERG 20
COMMON /SOL/ BGK(434, 86), ALOAD(434), ABGN MERG 30
COMMON /INT/ NEL, NJT, NMAT, NCLOAD, NPBC, NCON(182, 15), NBC(217, 2), NSTR MERG 40
1 ES, NGP, LM(12), LJ(12), NTLD, NTBY, NTIN, NLEL MERG 50
COMMON /FLPL/ COARD(217, 2), CLOAD(217, 2), ELCCN(10, 7), TITLE(10), ANGL MERG 60
1 (182), THNP(217) MERG 70
COMMON /INLAM/ LAMAT(50, 100), LYRNO(50), INDK(10) MERG 80
COMMON /FLLAM/ TKLAM(50, 100), JRLAM(50, 100) MERG 90
COMMON /COORD(12, 2), ELAST(3, 3), SS(36, 24), SN(36, 24), TK MERG 100
DIMENSION STK(24, 24), AK(24, 24), B(3, 24) MERG 110
REWIND 10 MERG 120
ZRC = 0.0D0 MERG 130
DC 1 I=1, 36 MERG 140
MERG 150

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```

C      DO 1 J=1,24
      SS(I,J) = ZRO
1      SA(I,J) = ZRO
C
C      DC 6 IK=1,NEL
C
C      DO 2 NN=1,NPEL
      LJT(NN) = NCON(IK,NN)
2      LM(NN) = (NCON(IK,NN)-1)*2
C
      NS = 3*NPEL
C
C      DO 3 I1=1,NPEL
      I2 = LJT(I1)
C
C      DO 3 J1=1,2
      COORD(I1,J1) = COORD(I2,J1)
C
      INCIC = NCON(IK,15)
      IF (INDIC.NE.1) GO TO 4
      CALL LAMC (IK)
      GO TO 5
4      THETA = ANGL(IK)
      MATN = NCON(IK,14)
      INDIC = INDK(MATN)
      CALL STFMAT (ELAST,MATN,THETA,INDIC,NSTRES)
      TK = ELCON(MATN,7)
      N = NCON(IK,13)*8
5      SET THICKNESS=1. FOR PLANE STRAIN
      IF (NSTRES.EQ.1) TK=1.0D0
      CALL QUAD5 (STK,AK,B,NS,N,NGP)
      WRITE (10) SS,SN
      IF (TK.EQ.0.0D0) TK=1.0D0
C
C      DO 6 I=1,NPEL
C
C      DC 6 J=1,NPEL
C
C      DO 6 K=1,2
      II = LM(I)+K
      KK = 2*(I-1)+K
C
C      DO 6 L=1,2
      JJ = LM(J)+L-II+1
      IF (JJ.LE.0) GO TO 6
      LL = 2*(J-1)+L

```



```

C      BGK(II,JJ) = BGK(II,JJ)+STK(KK,LL)*TK      650
6      CONTINUE      MERG 660
C      SUM = ZRO      MERG 670
C      CDN = ZRO      MERG 680
C      DC 7 I=1,NEQ      MERG 690
7      SUM = SUM+BGK(I,1)      MERG 700
      ABGN = SUM*1.0D20      MERG 710
      RETURN      MERG 720
      END      MERG 730
      SUBROUTINE LAMC (IK)      MERG 740
C      THIS SUBROUTINE CALCULATES THE STIFFNESS MATRIX FOR LAMINATED      MERG 750
C      COMPOSITE MATERIALS. THIS SUBROUTINE ASSUMES THAT THE LAMINATED      MERG 760
C      COMPOSITE IS OF BALANCED DESIGN SO THAT THE STIFFNESS AND BENDING      LAMC 10
C      PROPERTIES ARE UNCOUPLED. IT CALCULATES AN AVERAGE STIFFNESS MATRIX      LAMC 20
C      FROM THE STIFFNESS PROPERTIES OF EACH LAMINA.      LAMC 30
C      IK IS THE ELEMENT NUMBER      LAMC 40
C      IMPLICIT REAL*8(A-H,O-Z)      LAMC 50
C      IMPLICIT INTEGER*2(I-N)      LAMC 60
C      COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTR      LAMC 70
C      LES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL      LAMC 80
C      COMMON /FLPL/ COORD(217,2),CLOAD(217,2),ELCCN(10,7),TITLE(10),ANGL      LAMC 90
C      1(182),THNP(217)      LAMC 100
C      COMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)      LAMC 110
C      COMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)      LAMC 120
C      COMMON /COORD(12,2),ELAST(3,3),SS(36,24),SN(36,24),TK      LAMC 130
C      DIMENSION C(3,3)      LAMC 140
C      LAMTP = NCON(IK,14)      LAMC 150
C      THETA = ANGL(IK)      LAMC 160
C      TK = 0.0D0      LAMC 170
C      DC 1 I=1,3      LAMC 180
C      DC 1 J=1,3      LAMC 190
1      C(I,J) = 0.0D0      LAMC 200
C      LYRS = LYRNO(LAMTP)      LAMC 210
C      DC 3 I=1,LYRS      LAMC 220
C      MATN = LAMAT(LAMTP,I)      LAMC 230
C      PHI = THETA+ORLAM(LAMTP,I)      LAMC 240
C      INCIC = INDK(MATN)      LAMC 250
C      CALL STFMAT (ELAST,MATN,PHI,INDIC,NSTRES)      LAMC 260
C      TK = TK+TKLAM(LAMTP,I)      LAMC 270
C      DC 2 J=1,3      LAMC 280
C      END      LAMC 290
C      END      LAMC 300
C      END      LAMC 310
C      END      LAMC 320
C      END      LAMC 330
C      END      LAMC 340
C      END      LAMC 350
C      END      LAMC 360

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```

C      DC 2 K=1,3
C      2 C(J,K) = C(J,K)+ELAST(J,K)*TKLAM(LAMTP,I)
C      3 CCNTINUE
C
C      DO 4 J=1,3
C
C      DO 4 K=1,3
C      4 ELAST(J,K) = C(J,K)/TK
C      RETURN
C      END
C      SUBROUTINE STFMAT (C,MATN,THETA,INDIC,NSTRESS)
C      THIS SUBROUTINE ASSEMBLES THE ELASTIC STIFFNESS MATRIX FOR A HOOKEAN
C      ELEMENT. THE SUBROUTINE CALCULATES AND STORES IN C(3X3) THE VALUES
C      OF THE ELEMENTS OF THIS MATRIX FOR ISOTROPIC MATERIALS UNDER
C      EITHER PLANE STRAIN OR PLANE STRESS OR FOR ANISOTROPIC MATERIALS,
C      EXHIBITING TWO DIMENSIONAL ORTHOTROPY. FOR ANISOTROPIC MATERIALS,
C      THE ELEMENTS OF THIS MATRIX ARE FIRST COMPUTED IN THE MATERIAL
C      BASED L-T COORDINATE SYSTEM AND THEN TRANSFORMED TO THE PROBLEM
C      BASED X-Y COORDINATE SYSTEM.
C      IMPLICIT REAL*8(A-H,O-Z)
C      IMPLICIT INTEGER*2(I-N)
C      COMMON /FLPL/ CUARD(217,2), CLOAD(217,2), ELCON(10,7), TITLE(10), ANGL
C      1(182), THNP(217)
C      DIMENSION A(3,3), C(3,3)
C
C      DC 1 I=1,3
C
C      DO 1 J=1,3
C      A(I,J) = 0.000
C      C(I,J) = 0.000
C      1 CCNTINUE
C
C      IF (INCIC.EQ.1) GO TO 3
C      IF (NSTRES.EQ.1) GO TO 2
C      FORM THE ELASTIC STIFFNESS MATRIX FOR ISOTROPIC PLANE STRESS
C
C      E = ELCCN(MATN,1)
C      XNU = ELCON(MATN,2)
C      EO = E/(1.000-XNU*XNU)
C      C(1,1) = EO
C      C(1,2) = EO*XNU
C      C(2,1) = C(1,2)
C      C(2,2) = C(1,1)
C      C(3,3) = EO*(1.000-XNU)/2.000

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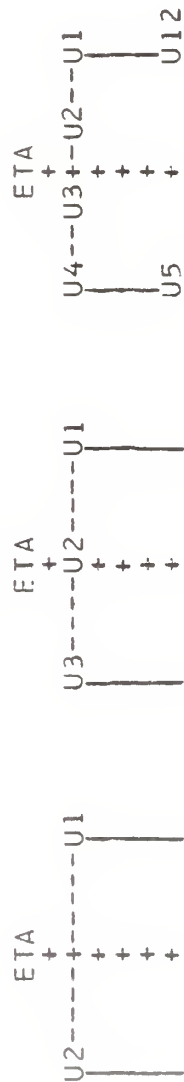
C      RETURN
C      2 CONTINUE
C      FORM THE ELASTIC STIFFNESS MATRIX FOR ISOTROPIC PLANE STRAIN
C
C      E = ELCCN(MATN,1)
C      XNU = ELCON(MATN,2)
C      EI = E/((1.000+XNU)*(1.000-2.000*XNU))
C      C(1,1) = EI*(1.000-XNU)
C      C(1,2) = EI*XNU
C      C(2,1) = C(1,2)
C      C(2,2) = C(1,1)
C      C(3,3) = EI*(0.500-XNU)
C      RETURN
C      3 CONTINUE
C      FORM THE ANISOTROPIC ORTHOTROPIC ELASTIC MATRIX IN THE MATERIAL BASED
C      L-T COORDINATE SYSTEM
C
C      EL = ELCON(MATN,1)
C      ET = ELCON(MATN,2)
C      XNUTL = ELCON(MATN,3)
C      XNUTL = ELCON(MATN,4)
C      GLT = ELCON(MATN,5)
C      IF (EL.LT.-0.000) EL=XNUTL*ET/XNUTL
C      IF (ET.LT.-0.000) ET=EL*XNUTL/XNUTL
C      IF (XNUTL.LT.-0.000) XNUTL=EL*XNUTL/ET
C      IF (XNUTL.LT.-0.000) XNUTL=XNUTL*ET/EL
C      Z = 1.000-XNUTL*XNUTL
C      A(1,1) = EL/Z
C      A(1,2) = ET*XNUTL/Z
C      A(2,1) = A(1,2)
C      A(2,2) = ET/Z
C      A(3,3) = GLT
C
C      THIS ARRAY IS NOW TRANSFORMED FROM THE MATERIAL L-T COORDINATE
C      SYSTEM TO THE PROBLEM X-Y COORDINATE SYSTEM. THE TRANSFORMATION IS
C      ACCOMPLISHED BY ROTATION ABOUT THE Z AXIS THROUGH THETA DEGREES.
C
C      PI = 3.1415926535900
C      THRAD = THETA*PI/180.000
C      CCS1 = DCOS(THRAD)
C      CCS2 = DCOS(THRAD)**2
C      CCS3 = DCOS(THRAD)**3
C      CCS4 = DCOS(THRAD)**4
C      SIN1 = DSIN(THRAD)
C      SIN2 = DSIN(THRAD)**2
C      SIN3 = DSIN(THRAD)**3

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SIN4 = DSIN(THRAD)**4
TWO = 2.0D0
FOUR = 4.0D0
C(1,1) = A(1,1)*COS4+TWO*(A(1,2)+TWO*A(3,3))*SIN2*COS2+A(2,2)*SIN4+COS4
C(1,2) = (A(1,1)+A(2,2)-FOUR*A(3,3))*SIN2*CCS2+A(1,2)*(SIN4+COS4)
C(1,3) = (A(1,1)-A(1,2)-TWO*A(3,3))*SIN1*COS3+(A(1,2)-A(2,2)+TWO*(
1A(3,3)))*SIN3*COS1
C(2,2) = A(1,1)*SIN4+TWO*(A(1,2)+TWO*A(3,3))*SIN2*COS2+A(2,2)*COS4
C(2,3) = (A(1,1)-A(1,2)-TWO*A(3,3))*SIN3*COS1+(A(1,2)-A(2,2)+TWO*A
1C(3,3))*SIN1*COS3
C(3,3) = (A(1,1)+A(2,2)-TWO*(A(1,2)+A(3,3)))*SIN2*COS2+A(3,3)*(SIN
14+COS4)
C(2,1) = C(1,2)
C(3,1) = C(1,3)
C(3,2) = C(2,3)
RETURN
END
SUBROUTINE QUAD5 (STK,AK,B,NS,N,NGP)
QUAD5 IS A NUMERICAL INTEGRATION SUBROUTINE THAT FORMS THE STIFFNESS
MATRIX FOR ANY OF THE THREE QUADRILATERAL ISOPARAMETRIC
ELEMENTS USED IN THIS PROGRAM.
GAUSSIAN QUADRATURE WITH A MAXIMUM OF FIVE ORDINATES IS USED.
STK IS AN ARRAY DIMENSIONED NXN THAT WILL CONTAIN THE STIFFNESS
MATRIX UPON RETURN TO THE CALLING PROGRAM
SS AND SN ARE THE STRESS AND STRAIN VERSUS NODAL POINT
DISPLACEMENT TRANSFORMATION MATRICES, BOTH ARE DIMENSIONED
NSXN WHERE NS= 3*NPT AND N= 2*NPT
AK IS AN ARRAY DIMENSIONED NXN REQUIRED IN FORMK
B IS AN ARRAY DIMENSIONED 2XN REQUIRED IN FORMK
NGP IS THE NUMBER OF GAUSS POINTS USED IN THE INTEGRATION
COORD(12,2) IS AN ARRAY CONTAINING THE COORDINATES OF ALL
THE NODAL POINTS OF THE ELEMENT TO BE CONSTRUCTED
ELAST(3,3) IS AN ARRAY CONTAINING ALL THE ELASTIC CONSTANTS
OF THE ELEMENT, THE ORDER OF THE STRESS COMPONENTS
IS: SIGMAX,SIGMAY,TAUXY

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```

2  X1(I) = X2(I)
   AI(I) = A2(I)
   GO TO 6
3  X1(I) = X3(I)
   AI(I) = A3(I)
   GO TO 6
4  X1(I) = X4(I)
   AI(I) = A4(I)
   GO TO 6
5  X1(I) = X5(I)
   AI(I) = A5(I)
6  CCNTINUE

      DO 7 I=1,NGP
      DC 7 J=1,NGP
7  AIA(I,J) = AI(I)*AI(J)

      DO 8 I=1,NGP
      X = XI(I)

      DC 8 J=1,NGP
      Y = XJ(J)
      CALL FORMK (AK,X,Y,B,N)
      AIAIJ = AIA(I,J)

      DC 8 K=1,N
      DC 8 L=1,N
8  STK(K,L) = STK(K,L)+AIAIJ*AK(K,L)

      DC 9 I=1,N
      DC 9 J=1,N
      STK(I,J) = (STK(I,J)+STK(J,I))*0.5D0
9  STK(J,I) = STK(I,J)

      NPT = N/2
      IGO = NPT/4

      DC 16 I=1,NPT
      GO TO (10,11,12), IGO
10 X = XYL(I,1)
   Y = XYL(I,2)
   GO TO 13

```



```

11 X = XYQ(I,1)
12 Y = XYQ(I,2)
13 GC TO 13
14 X = XYC(I,1)
15 Y = XYC(I,2)
16 CALL FORMK (AK,X,Y,B,N)
17
18 DO 14 L=1,3
19 DO 14 M=1,N
20 AK(L,M) = 0.000
21
22 DO 14 NN=1,3
23 AK(L,M) = AK(L,M)+ELAST(L,NN)*B(NN,M)
24
25 II = 3*(I-1)
26 DO 15 J=1,3
27 IJ = II+J
28
29 DO 15 K=1,N
30 SS(IJ,K) = AK(J,K)
31 SN(IJ,K) = B(J,K)
32
33 CCNTINUE
34
35 RETURN
36
37 SUBROUTINE FORMK (AK,X,Y,B,N)
38 FORMK FORMS THE STIFFNESS MATRIX AND THE B MATRIX AS FUNCTIONS
39 OF XI AND ETA FOR THE ISOPARAMETRIC ELEMENTS USED IN THIS PROGRAM
40
41 IMPLICIT REAL*8(A-H,O-Z)
42 IMPLICIT INTEGER*2(I-N)
43 DIMENSION AJ(2,2), AJIN(2,2), DNX(2,12), W1(2,12), B(3,24), B1(24,
44 13), AK(24,24)
45 CCMCN COORD(12,2),ELAST(3,3),SS(36,24),SN(36,24),TK
46 ZERO = 0.000
47
48 DO 1 I=1,3
49 DO 1 J=1,N
50 B(I,J) = ZERO
51
52 CNE = 1.000
53 TWO = 2.000
54 FOUR = 4.000
55 NPT = N/2
56
57 QUAD1280
58 QUAD1290
59 QUAD1300
60 QUAD1310
61 QUAD1320
62 QUAD1330
63 QUAD1340
64 QUAD1350
65 QUAD1360
66 QUAD1370
67 QUAD1380
68 QUAD1390
69 QUAD1400
70 QUAD1410
71 QUAD1420
72 QUAD1430
73 QUAD1440
74 QUAD1450
75 QUAD1460
76 QUAD1470
77 QUAD1480
78 QUAD1490
79 QUAD1500
80 QUAD1510
81 QUAD1520
82 QUAD1530
83 QUAD1540
84 QUAD1550
85 FRMK 10
86 FRMK 20
87 FRMK 30
88 FRMK 40
89 FRMK 50
90 FRMK 60
91 FRMK 70
92 FRMK 80
93 FRMK 90
94 FRMK 100
95 FRMK 110
96 FRMK 120
97 FRMK 130
98 FRMK 140
99 FRMK 150
100 FRMK 160
101 FRMK 170
102 FRMK 180
103 FRMK 190
104 FRMK 200

```

```

C FORM (2XNPT) MATRIX OF DERIV. OF THE INTERP. FUNCT. WRT XI AND ETA
C
C      IGC = N/8
C      GO TO (2,3,4), IGC
C
C      LINEAR FUNCTIONS
C
2  W1(1,3) = -(ONE-Y)/FOUR
   W1(1,4) = -W1(1,3)
   W1(1,1) = (ONE+Y)/FOUR
   W1(1,2) = -W1(1,1)
   W1(2,3) = -(ONE-X)/FOUR
   W1(2,4) = -(ONE+X)/FOUR
   W1(2,1) = -W1(2,4)
   W1(2,2) = -W1(2,3)
   GO TO 5
3  CONTINUE
C
C      QUADRATIC FUNCTIONS
C
C      TXPY = TWO*X+Y
C      TXMY = TWO*X-Y
C      TYPX = TWO*Y+X
C      TYMX = TWO*Y-X
C      CMY = ONE-Y
C      OPY = ONE+Y
C      OPX = ONE+X
C      OMX = ONE-X
C      W1(1,5) = OMY*TXPY/FOUR
C      W1(1,6) = -OMY*X
C      W1(1,7) = OMY*TXMY/FOUR
C      W1(1,8) = OPY*OMY/TWO
C      W1(1,1) = OPY*TXPY/FOUR
C      W1(1,2) = -OPY*X
C      W1(1,3) = OPY*TXMY/FOUR
C      W1(1,4) = -OPY*OMY/TWO
C      W1(2,5) = OMX*TYPX/FOUR
C      W1(2,6) = -OPX*OMX/TWO
C      W1(2,7) = OPX*TYMX/FOUR
C      W1(2,8) = -Y*OPX
C      W1(2,1) = OPX*TYPX/FOUR
C      W1(2,2) = OPX*OMX/TWO
C      W1(2,3) = OMX*TYMX/FOUR
C      W1(2,4) = -Y*OMX
C      GO TO 5
4  CONTINUE
C

```

```

FRMK 210
FRMK 220
FRMK 230
FRMK 240
FRMK 250
FRMK 260
FRMK 270
FRMK 280
FRMK 290
FRMK 300
FRMK 310
FRMK 320
FRMK 330
FRMK 340
FRMK 350
FRMK 360
FRMK 370
FRMK 380
FRMK 390
FRMK 400
FRMK 410
FRMK 420
FRMK 430
FRMK 440
FRMK 450
FRMK 460
FRMK 470
FRMK 480
FRMK 490
FRMK 500
FRMK 510
FRMK 520
FRMK 530
FRMK 540
FRMK 550
FRMK 560
FRMK 570
FRMK 580
FRMK 590
FRMK 600
FRMK 610
FRMK 620
FRMK 630
FRMK 640
FRMK 650
FRMK 660
FRMK 670
FRMK 680

```



# CUBIC FUNCTIONS

C

```

CPIX = ONE+X
OPY = ONE+Y
CMX = ONE-X
CMY = ONE-Y
TPTMNX = 3.0D0+2.0D0*X-9.0D0*X*X
TMTMNX = 3.0D0-2.0D0*X-9.0D0*X*X
TPTMNY = 3.0D0+2.0D0*Y-9.0D0*Y*Y
TMTMNY = 3.0D0-2.0D0*Y-9.0D0*Y*Y
TYPO = 3.0D0*Y+ONE
TYMO = 3.0D0*Y-ONE
TXPO = 3.0D0*X+ONE
TXMO = 3.0D0*X-ONE
TMTPEX = 10.0D0-27.0D0*X*X+18.0D0*X-9.0D0*Y*Y
TMTMEY = 10.0D0-27.0D0*X*X-18.0D0*X-9.0D0*Y*Y
TMNPEX = 10.0D0-9.0D0*X*X+18.0D0*Y-27.0D0*Y*Y
TMNMEY = 10.0D0-9.0D0*X*X-18.0D0*Y-27.0D0*Y*Y
TTT = 32.0D0
TTN = 32.0D0/9.0D0
WI(1,7) = OMY*TMPEX/TTT
WI(1,8) = -TPTMNX*OMY/TTN
WI(1,9) = TMTMNX*OMY/TTN
WI(1,10) = -OMY*TMMEY/TTT
WI(1,11) = -OPY*OMY*TYMO/TTN
WI(1,12) = OPY*OMY*TYPO/TTN
WI(1,1) = -OPY*TMMEY/TTT
WI(1,2) = TMTMNX*OPY/TTN
WI(1,3) = -TPTMNX*OPY/TTN
WI(1,4) = OPY*TMPEX/TTT
WI(1,5) = -OPY*OMY*TYPO/TTN
WI(1,6) = OPY*OMY*TYMO/TTN
WI(2,7) = OMX*TMNPEY/TTT
WI(2,8) = OPX*OMX*TXMO/TTN
WI(2,9) = -OPX*OMX*TXPO/TTN
WI(2,10) = OPX*TMNPEY/TTT
WI(2,11) = -TPTMNY*OPX/TTN
WI(2,12) = TMTMNY*OPX/TTN
WI(2,1) = -OPX*TMNMEY/TTT
WI(2,2) = OPX*OMX*TXPO/TTN
WI(2,3) = -OPX*OMX*TXMO/TTN
WI(2,4) = -OMX*TMNMEY/TTT
WI(2,5) = OMX*TMIMNY/TTN
WI(2,6) = -OMX*TPTMNY/TTN

```

5 CCNTINUE

C FORM JACOBIAN OF THE TRANSFORMATION IN AJ(I,J)

C

FRMK 690  
FRMK 700  
FRMK 710  
FRMK 720  
FRMK 730  
FRMK 740  
FRMK 750  
FRMK 760  
FRMK 770  
FRMK 780  
FRMK 790  
FRMK 800  
FRMK 810  
FRMK 820  
FRMK 830  
FRMK 840  
FRMK 850  
FRMK 860  
FRMK 870  
FRMK 880  
FRMK 890  
FRMK 900  
FRMK 910  
FRMK 920  
FRMK 930  
FRMK 940  
FRMK 950  
FRMK 960  
FRMK 970  
FRMK 980  
FRMK 990  
FRMK 1000  
FRMK 1010  
FRMK 1020  
FRMK 1030  
FRMK 1040  
FRMK 1050  
FRMK 1060  
FRMK 1070  
FRMK 1080  
FRMK 1090  
FRMK 1100  
FRMK 1110  
FRMK 1120  
FRMK 1130  
FRMK 1140  
FRMK 1150  
FRMK 1160

```

C      DC 6 I=1,2
C
C      DO 6 J=1,2
C      AJ(I,J) = ZERO
C
C      DO 6 K=1,NPT
C      6 AJ(I,J) = AJ(I,J)+WL(I,K)*COORD(K,J)
C
C      CALCULATE DETERMINANT OF THE JACOBIAN
C      DTJ = AJ(1,1)*AJ(2,2)-AJ(1,2)*AJ(2,1)
C      INVERT JACOBIAN IN AJIN
C      AJIN(1,1) = AJ(2,2)/DTJ
C      AJIN(1,2) = -AJ(1,2)/DTJ
C      AJIN(2,1) = -AJ(2,1)/DTJ
C      AJIN(2,2) = AJ(1,1)/DTJ
C
C      FORM (2XNPT) MATRIX OF DERIVATIVES OF INTERP. FUNCT WRT X AND Y
C
C      DC 7 I=1,2
C      DO 7 J=1,NPT
C      DNX(I,J) = ZERO
C
C      DC 7 K=1,2
C      7 DNX(I,J) = DNX(I,J)+AJIN(I,K)*WL(K,J)
C
C      FORM B(I,J) MATRIX
C
C      DC 8 I=1,NPT
C      IOO = 2*I-1
C      IEV = 2*I
C      B(1,IOO) = DNX(1,I)
C      B(2,IEV) = DNX(2,I)
C      B(3,IEV) = DNX(1,I)
C      8 B(3,IOO) = DNX(2,I)
C
C      FORM STIFFNESS BY THE CONGRUENT TRANSFORMATION BT*D*B
C

```

```

FRMK1170
FRMK1180
FRMK1190
FRMK1200
FRMK1210
FRMK1220
FRMK1230
FRMK1240
FRMK1250
FRMK1260
FRMK1270
FRMK1280
FRMK1290
FRMK1300
FRMK1310
FRMK1320
FRMK1330
FRMK1340
FRMK1350
FRMK1360
FRMK1370
FRMK1380
FRMK1390
FRMK1400
FRMK1410
FRMK1420
FRMK1430
FRMK1440
FRMK1450
FRMK1460
FRMK1470
FRMK1480
FRMK1490
FRMK1500
FRMK1510
FRMK1520
FRMK1530
FRMK1540
FRMK1550
FRMK1560
FRMK1570
FRMK1580
FRMK1590
FRMK1600
FRMK1610
FRMK1620
FRMK1630
FRMK1640

```

```

C          DC 9 I=1,N
C          DO 9 J=1,3
C          B1(I,J) = ZERO
C
C          DO 9 K=1,3
C          9 B1(I,J) = B1(I,J)+B(K,I)*ELAST(K,J)
C
C          DC 10 I=1,N
C          DO 10 J=1,N
C          AK(I,J) = ZERO
C
C          DC 10 K=1,3
C          10 AK(I,J) = AK(I,J)+B1(I,K)*B(K,J)
C
C          DC 11 I=1,N
C          DC 11 J=1,N
C          AK(I,J) = ((AK(I,J)+AK(J,I))/2.0D0)*DTJ
C          11 AK(J,I) = AK(I,J)
C
C          RETURN
C          END
C          SUBROUTINE LDLT (N,M)
C          *****
C          ***** THIS SUBROUTINE DECOMPOSES THE COEFFICIENT MATRIX A* OF THE LINEAR
C          ***** Banded Symmetrical System (A*)(X) = (B), INTO THE PRODUCT (L)(D)(LT)
C          ***** THE UPPER BAND ONLY IS USED, AND IT MUST BE STORED IN A RECTANGULAR
C          ***** ARRAY (A) WITH ALL ITS DIAGONAL ELEMENTS IN THE FIRST COLUMN
C          ***** MATRIX (A) IS DESTROYED IN THE PROCESS AND THE RESULTS RETURNED IN
C          ***** THE SAME ARRAY. MATRIX (D) IS PUT IN THE FIRST COLUMN AND THE REST
C          ***** OF THE MATRIX CONTAINS THE ELEMENTS OF THE UNIT TRIANGULAR MATRIX
C          ***** IN Banded Form WITHOUT ITS IMPLIED UNIT DIAGONAL ELEMENTS.
C
C          A IS THE NAME OF THE COEFFICIENT MATRIX
C          N IS THE NUMBER OF EQUATIONS IN THE SYSTEM
C          M IS THE HALF BAND WIDTH OF THE SYSTEM
C
C          CODED BY GILLES CANTIN, NAVAL POSTGRADUATE SCHOOL, MAY 1972
C          *****
C          ***** IMPLICIT REAL*8 (A-H,O-Z)
C          *****
C          ***** IMPLICIT INTEGER*2 (I-N)
C          *****
C          ***** COMMON /SOL/ A(434,86),B(434),ABGN
C          *****
C          ***** AN = N
C          *****
C          ***** APZRO = ABGN*1.0D-30/AN
C          *****
FRMK1650
FRMK1660
FRMK1670
FRMK1680
FRMK1690
FRMK1700
FRMK1710
FRMK1720
FRMK1730
FRMK1740
FRMK1750
FRMK1760
FRMK1770
FRMK1780
FRMK1790
FRMK1800
FRMK1810
FRMK1820
FRMK1830
FRMK1840
FRMK1850
FRMK1860
FRMK1870
FRMK1880
FRMK1890
FRMK1900
FRMK1910
LDLT 10
LDLT 20
LDLT 30
LDLT 40
LDLT 50
LDLT 60
LDLT 70
LDLT 80
LDLT 90
LDLT 100
LDLT 110
LDLT 120
LDLT 130
LDLT 140
LDLT 150
LDLT 160
LDLT 170
LDLT 180
LDLT 190
LDLT 200
LDLT 210
LDLT 220

```

```

C      AVSN = APZRO/ABGN
      ZRC = 0.000
      NM = N-1
      DC 3 I=1,NM
      DIAG = A(I,1)
      IF (DIAG.EQ.ZRO) GO TO 4
      IF (DIAG.LE.APZRO) WRITE (6,6) I
C
C      DO 1 J=2,M
      IF (DABS(A(I,J)).LE.AVSN) A(I,J)=0.000
      1 A(I,J) = A(I,J)/DIAG
C
C      DC 3 J=2,M
      L = I+J-1
      IF (L.GT.N) GO TO 3
      AA = A(I,J)*DIAG
      IF (AA.EQ.ZRO) GO TO 3
C
C      DO 2 K=J,M
      ML = 1+K-J
      2 A(L,ML) = A(L,ML)-AA*A(I,K)
      3 CCNTINUE
C
      RETURN
      4 WRITE (6,5) I
      STOP
C
      5 FORMAT (//,5X,' MATRIX IS SINGULAR ',1I5)
      6 FORMAT (//,5X,' WARNING MATRIX IS NEARLY SINGULAR EXFCUTION CONTL
      1 INUES ', 1I5,/,5X,' RESIDUAL ITERATION MAY YIELD ERRONEOUS RESULT
      2 ,//)
      END
      SUBROUTINE SOLV (N,M)
      *****
      THIS SUBROUTINE PERFORMS A FORWARD SUBSTITUTION, FOLLOWED BY A
      BACKWARD SUBSTITUTION FOR THE SYSTEM (A*) X=(B), AFTER THE
      MATRIX (A) HAS BEEN DECOMPOSED BY THE SUBROUTINE LDLT
C
C      A IS THE NAME OF THE DECOMPOSED MATRIX CF COEFFICIENT
C      B IS THE NAME OF THE COLUMN LOAD VECTOR OR RIGHT HAND MEMBER
C      THE ANSWERS ARE RETURNED IN THIS VECTOR UPON EXIT
C      N IS THE NUMBER OF EQUATIONS IN THE SYSTEM
C      M IS THE HALF BAND WIDTH OF THE SYTEM
C
      AFTER ONE CALL TO LDLT, ONE CALL OF SOLV PER LOAD VECTOR IS REQ'D.
      *****
      SOLV 10
      SOLV 20
      SOLV 30
      SOLV 40
      SOLV 50
      SOLV 60
      SOLV 70
      SOLV 80
      SOLV 90
      SOLV 100
      SOLV 110
      SOLV 120
      SOLV 130

```

```

C      CODED BY GILLES CANTIN, NAVAL POSTGRADUATE SCHOOL, MAY 1972
C      *****
C      IMPLICIT REAL*8(A-H,O-Z)
C      IMPLICIT INTEGER*2(I-N)
C      COMMON /SOL/ A(434,86),B(434),ABGN
C      NM = N-1
C
C      DC 1 I=1,NM
C      BI = B(I)
C
C      DC 1 J=2,M
C      L=I+J-1
C      IF (L.GT.N) GO TO 1
C      B(L) = B(L)-A(I,J)*BI
C      1 CCNTINUE
C
C      DC 2 I=1,N
C      2 B(I) = B(I)/A(I,1)
C
C      DC 3 L=2,N
C      IR = N-L+1
C      BIRP = B(IR+1)
C
C      DC 3 J=2,M
C      IRO = IR-J+2
C      IF (IRO.LE.0) GO TO 3
C      B(IRO) = B(IRO)-A(IRO,J)*BIRP
C      3 CCNTINUE
C
C      RETURN
C      END
C      SUBROUTINE STRESS (NPCL)
C      IMPLICIT REAL*8(A-H,O-Z)
C      IMPLICIT INTEGER*2(I-N)
C      COMMON /SOL/ BGK(434,86),DISP(434),ABGN
C      COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTR
C      1 ES,NGP,LM(12),LJT(12),NTLD,NTBY,NTIN,NLEL
C      COMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),ANGL
C      1(182),THNP(217)
C      COMMON COORD(12,2),ELAST(3,3),SS(36,24),SN(36,24),TK
C      DIMENSION SSJNT(217,7), SNJNT(217,7), SSEL(36), SNEP(36), DSPEL(36
C      1), DIMENSION TEEL(12)
C      EQUIVALENCE (BGK(1,1),SSJNT(1,1),SSJNT(1,5),SNJNT(1,1)), (BGK(1,9)
C      1),SSEL(1)), (BGK(1,10),SNEP(1)), (BGK(1,11),DSPEL(1))
C      PI = 3.14159265359D0

```

STRS 160  
STRS 170  
STRS 180  
STRS 190  
STRS 200  
STRS 210  
STRS 220  
STRS 230  
STRS 240  
STRS 250  
STRS 260  
STRS 270  
STRS 280  
STRS 290  
STRS 300  
STRS 310  
STRS 320  
STRS 330  
STRS 340  
STRS 350  
STRS 360  
STRS 370  
STRS 380  
STRS 390  
STRS 400  
STRS 410  
STRS 420  
STRS 430  
STRS 440  
STRS 450  
STRS 460  
STRS 470  
STRS 480  
STRS 490  
STRS 500  
STRS 510  
STRS 520  
STRS 530  
STRS 540  
STRS 550  
STRS 560  
STRS 570  
STRS 580  
STRS 590  
STRS 600  
STRS 610  
STRS 620  
STRS 630

```

ZRC = 0.0D0
REWIND IO
DC 1 I=1,NJT
DO 1 J=1,7
  SJJNT(I,J) = ZRO
  1 SSJNT(I,J) = ZRO
C
C
      DO 11 I=1,NEL
      READ (IO) SS,SN
      IF (NTLD.EQ.0) GO TO 3
      MATN = NCON(I,14)
      YMOD = ELCON(MATN,1)
      PRAT = ELCON(MATN,2)
      ALPH = ELCON(MATN,3)
      IF (NSTRES.EQ.0) GO TO 2
      CCNST = ALPH*YMOD/(1.0D0-2.0D0*PRAT)
      GO TO 3
      2 CCNST = ALPH*YMOD/(1.0D0-PRAT)
      3 CONTINUE
C
      DO 4 J=1,NPEL
      NCIJ = NCON(I,J)
      JA = NCIJ*2-1
      JJ = J*2-1
      IF (NTLD.NE.0) TEEL(J)=THNP(NCIJ)
      DSPEL(JJ) = DISP(JA)
      JJ = JJ+1
      JA = JA+1
      4 DSPEL(JJ) = DISP(JA)
C
      NJLM = NPEL*2
      NPELT = NPEL*3
C
      DC 5 I1=1,NPELT
      SSEL(I1) = ZRO
      SNEL(I1) = ZRO
C
      DO 5 J1=1,NJLM
      SSEL(I1) = SSEL(I1)+SS(I1,J1)*DSPEL(J1)
      SNEL(I1) = SNEL(I1)+SN(I1,J1)*DSPEL(J1)
      5
      IF (NTLD.EQ.0) GO TO 7
C
      DO 6 J=1,NPEL
      JJ0 = J*3-2

```

```

JJD = JJO+1
CCRT = CONST*TEEL(J)
SSEL(JJO) = SSEL(JJG)-CORT
SSEL(JJD) = SSEL(JJD)-CORT
6 CCNTINUE
C
7 CONTINUE
IF (NMAT.EQ.1) GO TO 9
WRITE (6,19) I
ILO = 0
IUP = 0
C
DC 8 J=1,NPEL
IJT = NCON(I,J)
ILC = IUP+1
IUP = ILO+2
8 WRITE (6,18) IJT,(SSEL(K),K=ILO,IUP),(SNEL(L),L=ILO,IUP)
C
9 CCNTINUE
ICT = 0
C
DC 11 I2=1,NPEL
I2A = NCON(I,I2)
C
DC 10 J2=1,3
ICT = ICT+1
SSJNT(I2A,J2) = SSEL(ICT)+SSJNT(I2A,J2)
10 SAJNT(I2A,J2) = SNEL(ICT)+SNJNT(I2A,J2)
C
11 SAJNT(I2A,7) = SNJNT(I2A,7)+1.
C
C
DC 15 I3=1,NJT
SCNT = SNJNT(I3,7)
C
DC 12 J3=1,3
SSJNT(I3,J3) = SSJNT(I3,J3)/SCNT
12 SAJNT(I3,J3) = SNJNT(I3,J3)/SCNT
C
GAU = SSJNT(I3,3)
XPSY = (SSJNT(I3,1)+SSJNT(I3,2))/2.0D0
XMSY = (SSJNT(I3,1)-SSJNT(I3,2))/2.0D0
AINT = DSQRT(XMSY*XMSY+GAU*GAU)
SSJNT(I3,4) = XPSY+AINT
SSJNT(I3,5) = XPSY-AINT
SSJNT(I3,6) = (SSJNT(I3,4)-SSJNT(I3,5))/2.0D0
IF (DABS(XMSY).LE.1.D-10) GO TO 13
RAN = DATAN2(GAU,XMSY)

```







BCND 130  
BCND 140  
BCND 150  
BCND 160  
BCND 170  
BCND 180  
BCND 190  
BCND 200  
BCND 210  
BCND 220  
BCND 230  
BCND 240  
BCND 250  
BCND 260  
BCND 270  
BCND 280  
BCND 290  
BCND 300  
BCND 310  
BCND 320  
BCND 330  
BCND 340  
BCND 350  
BCND 360  
BCND 370  
BCND 380  
BCND 390  
BCND 400  
BCND 410  
BCND 420  
BCND 430  
BCND 440  
BCND 450  
BCND 460  
BCND 470  
BCND 480  
BCND 490  
BCND 500  
BCND 510  
BCND 520  
BCND 530  
BCND 540  
BCND 550  
BCND 560  
BCND 570  
BCND 580  
BCND 590  
BCND 600

```

IJT = NBC(I,1)
IEQ = IJT*2-1
KCODE = NBC(I,2)
ICIA = 0
GC TO (1,3,4,6,10,11,13), KODE
1 IEQ = IEQ+1
2 BGK(IEQ,1) = ABGN
  IF (IDIA.EQ.1) GO TO 5
  GC TO 16
3 GC TO 2
4 IDIA = 1
5 IDIA = 0
  GO TO 1 CLOAD(IJT,2)
6 CXY = CLOAD(IJT,2)
  IEQ = IEQ+1
7 ALOAD(IEQ) = ALOAD(IEQ)-BGK(IEQ,1)*DXY
  BGK(IEQ,1) = ABGN
C
  DC 9 J=2,NBAND
  IROW = IEQ-J+1
  ICOL = IEQ-1+J
  IF (IROW.LT.1) GO TO 8
  ALOAD(IROW) = ALOAD(IROW)-BGK(IROW,J)*DXY
8 IF (ICOL.GT.NEQ) GO TO 9
  ALOAD(ICOL) = ALOAD(ICOL)-BGK(IEQ,J)*DXY
9 CCNTINUE
C
  IF (IDIA.EQ.1) GO TO 12
  GC TO 16
10 DXY = CLOAD(IJT,1)
  GO TO 7
11 ICIA = 1
  DXY = CLOAD(IJT,1)
  GC TO 7
12 ICIA = 0
  GC TO 6
13 ALPHA = CLOAD(IJT,2)*PI/180.0D0
  CAL = DCOS(ALPHA)
  SAL = CSIN(ALPHA)
  IECP = IEQ+1
C
  DC 15 J=3,NBAND
  IROW = IEQ-J+2
  JM = J-1
  IF (IROW.LT.1) GO TO 14
  A1 = BGK(IROW,JM)*CAL+BGK(IROW,J)*SAL
  A2 = BGK(IROW,J)*CAL-BGK(IROW,JM)*SAL

```

```

      BGK(IROW,JM) = A1
      BGK(IROW,J) = A2
14  IF ((IEQ+J-1).GT.NEQ) GO TO 15
      A3 = BGK(IEQ,J)*CAL+BGK(IEQP,JM)*SAL
      A4 = BGK(IEQP,JM)*CAL-BGK(IEQ,J)*SAL
      BGK(IEQ,J) = A3
      BGK(IEQP,JM) = A4
15  CCNTINUE
C
      A1 = BGK(IEQ,1)*CAL*CAL+BGK(IEQ,2)*CAL*SAL*2.0+BGK(IEQP,1)*SAL*SAL
      A2 = BGK(IEQ,2)*((CAL*CAL-SAL*SAL)+(BGK(IEQP,1)-BGK(IEQ,1))*SAL*CAL
      BGK(IEQ,1) = A1
      BGK(IEQ,2) = A2
      BGK(IEQP,1) = ABGN
16  CCNTINUE
C
      RETURN
      END
      SUBROUTINE DISPL(A-H,O-Z)
      IMPLICIT REAL*8(A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      CCMMON /SOL/ BGK(434,86), DISP(434), ABGN
      CCMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTRD
      IES,NGP,LM(12),LJT(12),NTLO,NTBY,NTIN,NLEL
      CCMMON /FLPL/ COARD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),ANGLD
      I(182),THNP(217)
      CCMMON /MDIM/ MEL,MJT,MMT,MBD,MLEL,MLYR
      DIMENSION REACT(218,2)
      EQUIVALENCE (BGK(1,1),REACT(1,1))
      PI = 3.1415926535900
C
      DO 11 I=1,NPBC
      IJT = NBC(I,1)
      IEQ = IJT*2-1
      KODE = NBC(I,2)
      ICIA = 0
      DPBC = 0.0D0
      GO TO (1,3,4,6,7,8,10), KODE
      1 IEQ = IEQ+1
      JR = 2
      2 REACT(I,JR) = -DISP(IEQ)*ABGN
      DISP(IEQ) = DPBC
      IF (IDIA.EQ.1) GO TO 5
      IF (IDIA.EQ.2) GO TO 9
      GO TO 11
      3 JR = 1
      GO TO 2
      4 IDIA = 1

```

```

      JR = 1
      GO TO 2
5     ICIA = 0
      GC TO 1
6     DPBC = CLOAD(IJT,2)
      GO TO 1
7     JR = 1
      DPBC = CLOAD(IJT,1)
      GC TO 2
8     ICIA = 2
      JR = 1
      DPBC = CLOAD(IJT,1)
      GC TO 2
9     ICIA = 0
      DPBC = CLOAD(IJT,2)
      GO TO 1
10    ALPHA = CLOAD(IJT,2)*PI/180.000
      CAL = DCOS(ALPHA)
      SAL = DSIN(ALPHA)
      A1 = DISP(IEQ)*CAL
      A2 = DISP(IEQ)*SAL
      IEQP = IEQ+1
      A3 = -DISP(IEQP)*SAL*ABGN
      A4 = DISP(IEQP)*CAL*ABGN
      DISP(IEQ) = A1
      DISP(IEQP) = A2
      REACT(I,1) = -A3
      REACT(I,2) = -A4
11    CGCONTINUE
      C
      MJP = MJT+1
      REACT(MJP,1) = 0.000
      REACT(MJP,2) = 0.000
      C
      DC 12 I=1,NPBC
      REACT(MJP,1) = REACT(MJP,1)+REACT(I,1)
12    REACT(MJP,2) = REACT(MJP,2)+REACT(I,2)
      C
      RETURN
      END

```

DISP 310  
DISP 320  
DISP 330  
DISP 340  
DISP 350  
DISP 360  
DISP 370  
DISP 380  
DISP 390  
DISP 400  
DISP 410  
DISP 420  
DISP 430  
DISP 440  
DISP 450  
DISP 460  
DISP 470  
DISP 480  
DISP 490  
DISP 500  
DISP 510  
DISP 520  
DISP 530  
DISP 540  
DISP 550  
DISP 560  
DISP 570  
DISP 580  
DISP 590  
DISP 600  
DISP 610  
DISP 620  
DISP 630  
DISP 640  
DISP 650  
DISP 660  
DISP 670  
DISP 680  
DISP 690  
DISP 700

```

PROGRAM ISLOAD
*****
THIS SERVICE PROGRAM FORMS CONSISTENT LOAD VECTORS FOR ANY OF THE
FINITE ELEMENTS OF THE PROGRAM ISANIS
*****
THREE DIFFERENT LOAD VECTORS CAN BE FORMED.
(A) THERMAL LOAD CORRESPONDING TO A TEMPERATURE DISTRIBUTION
(B) BODY FORCE LOAD/UNIT VOLUME DISTRIBUTED THROUGH AN ELEMENT
(C) INITIAL STRETCH OR CONTRACTION OF AN ELEMENT
*****
THE LOAD INTENSITY FUNCTION OVER AN ELEMENT IS DETERMINED BY
ITS NODAL VALUES AND THE SAME SHAPE FUNCTIONS USED IN THE
CONSTRUCTION OF THE ELEMENT ITSELF. THUS A 4 NODDED ELEMENT
WILL ALLOW LINEAR VARIATION, AN 8 NODDED ELEMENT QUADRATIC VARIATION
AND A 12 NODDED ELEMENT CUBIC VARIATION
*****
THIS PROGRAM IS AN EXPANSION OF THE PLISOP LOAD GENERATOR
CODED BY GILLES CANTIN IN 1972
JAMES M. GILL, MARCH 1975
*****
INPUT CARDS NEEDED
*****
TITLE (20A4) ONE CARD PER PROBLEM
*****
PROBLEM PARAMETERS (12I5) ONE CARD PER PROBLEM
*****
COL: VARIABLE
1-5 NEL NUMBER OF ELEMENTS (MAX. IS 182)
6-10 NJT NUMBER OF JOINTS (MAX. IS 217)
11-15 NMAT NUMBER OF MATERIALS (MAX. IS 10)
16-20 NCLoad NUMBER OF CONCENTRATED LOADS
21-25 NPBC NUMBER OF JOINTS WITH BOUNDARY CONDITIONS
26-30 NSTRES ISOTROPIC MATERIAL PLANE STRESSES=0 MEANS
PLANE STRESS, NSTRES=1 MEANS PLANE STRAIN
31-35 NGP NUMBER OF GAUSS POINTS DESIRED IN THE INTEGRATION
36-40 NTLd 1 IF A THERMAL LOAD IS PRESENT 0 OTHERWISE
41-45 NTBty 1 IF A BODY FORCE LOAD IS PRESENT 0 OTHERWISE
46-50 NTIN 1 IF AN INITIAL STRETCH IS TO BE CONSIDERED
51-55 NCARD MUST BE 1 IF A PUNCHED LOAD VECTOR SUITABLE FOR
USE WITH ISANIS IS DESIRED
*****
NLEL NUMBER OF LAMINATE TYPES WHOSE MATERIAL
PROPERTIES ARE TO BE CALCULATED FROM
INPUT LAMINA PROPERTIES. (MAX. 50)
*****
(15,5X,4G15.8,I5) NJT CARDS PER PROBLEM
*****
JOINT CARDS
*****

```

COLUMN	VARIABLE	MAIN
1-5	JOINT NUMBER	450
11-25	X COORDINATE OF THE JOINT (CARTESIAN COORD)	MAIN 460
	RADIUS VECTOR LENGTH (POLAR COORD.)	MAIN 470
26-40	Y-COORDINATE OF THE JOINT (CARTESIAN COORD)	MAIN 480
	POLAR ANGLE IN DEGREES (POLAR COORD.)	MAIN 490
41-55	X-COMPONENT OF THE LOAD AT THE JOINT (CART)	MAIN 500
56-70	Y-COMPONENT OF THE LOAD AT THE JOINT (CART)	MAIN 510
70-75	COORDINATE SYSTEM INDICATOR. IND. = ZERO IF MAIN	520
	THE COORDINATE SYSTEM IS CARTESIAN, IF IND	530
	IS NOT ZERO, THE COORDINATE SYSTEM IS POLAR	540
		550
		560
MATERIAL	PROPERTIES CARDS(215,7F10.2)	MAIN 570
COLUMN	VARIABLE	MAIN 580
1-5	MATERIAL NUMBER	MAIN 590
6-10	INDK=0 FOR ISOTROPIC, INDK=1 FOR ORTHOTROPIC	MAIN 600
11-20	YOUNG'S MODULUS (ISOTROPIC MATERIAL)	MAIN 610
	E(L) (ORTHOTROPIC MATERIAL)	MAIN 620
21-30	POISSON'S RATIO (ISOTROPIC MATERIAL)	MAIN 630
	E(T) (ORTHOTROPIC MATERIAL)	MAIN 640
31-40	NOT USED (ISOTROPIC MATERIAL)	MAIN 650
	NU(LT) (ORTHOTROPIC MATERIAL)	MAIN 660
41-50	NOT USED (ISOTROPIC MATERIAL)	MAIN 670
	NU(TL) (ORTHOTROPIC MATERIAL)	MAIN 680
	FOR ORTHOTROPIC MATERIALS, ONLY 3 OF THE ABOVE 4 ENGIN.	MAIN 690
	CONSTANTS (E(L), E(T), NU(LT), NU(TL)) NEED BE KNOWN. FOR AN	MAIN 700
	UNKNOWN CONSTANT ENTER -1.0 IN THE APPROPRIATE COLUMN.	MAIN 710
51-60	NOT USED (ISOTROPIC MATERIALS)	MAIN 720
	G(LT) (ORTHOTROPIC MATERIALS)	MAIN 730
61-70	ALPHA COEFF. OF ISOTROPIC THERMAL EXPANSION	MAIN 740
	NOT USED FOR THERMALLY ANISOTROPIC MATERIALS	MAIN 750
71-80	MATERIAL THICKNESS. THIS IS USED FOR ELEMENTS	MAIN 760
	WHOSE PROPERTIES ARE NOT TO BE CALCULATED FROM	MAIN 770
	INPUT LAMINA PROPERTIES.	MAIN 780
		MAIN 790
ELEMENT	CARDS	MAIN 800
1-4	(15I4,F10.5,I5) NEL CARDS PER PROBLEM	MAIN 810
5-52	ELEMENT IDENTIFICATION NUMBER	MAIN 820
53-56	ELEMENT CONNECTIVITY (COUNTER-CLOCKWISE)	MAIN 830
	ELEMENT KIND - USE	MAIN 840
	1 FOR A FOUR NODAL POINT ELEMENT	MAIN 850
	2 FOR AN EIGHT NODAL POINT ELEMENT	MAIN 860
	3 FOR A TWELVE NODAL POINT ELEMENT	MAIN 870
57-60	MATERIAL/LAMINATE IDENTIFICATION NUMBER	MAIN 880
61-70	THETA (DEGREES) USED FOR ORTHOTROPIC MATERIALS	MAIN 890
	THETA IS THE ANGLE FROM THE PROBLEM BASED X AXIS	MAIN 900
	TO THE MATERIAL BASED L AXIS	MAIN 910
71-75	ENTER 1 FOR A LAMINATED ELEMENT WHOSE	MAIN 920
	PROPERTIES ARE TO BE CALCULATED FROM	





```

C      6-20      TEMPERATURE AT THE NODE
C      21-35      X COMP. OF BODY FORCE INTENSITY AT THE NODE
C      36-50      Y COMP. OF BODY FORCE INTENSITY AT THE NODE
C      51-65      X COMP. OF INITIAL DISP. AT THE NODE
C      66-80      Y COMP. OF INITIAL DISP. AT THE NODE
C
C      AS MANY PROBLEMS AS ONE DESIRES MAY BE STACKED, BUT THE LAST CARD OF
C      A RUN MUST CONTAIN THE WORD STOP IN THE FIRST FOUR COLUMNS
C
C      IMPLICIT REAL*8(A-H,O-Z)
C      IMPLICIT INTEGER*2(I-N)
C      COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTR
1     IES,NGP,LM(12),LJT(12),NLEL
C      COMMON /MDIM/ MEL,MJT,MMT,MBD,MLEL,MLYR
C      COMMON /FLPL/ COARD(217,2),TLOAD(217,2),ELCON(10,7),TITLE(10),THNP
1     I(217),BYNP(217,2),DPIN(217,2),ANGL(182)
C      COMMON /LOAD/ TEMP(12),BDFY(12,2),DSPN(12,2),TK,ALPHA,NN,NTLD,NTBY
1     NTIN,NCARD
C      COMMON /CC/ COORD(12,2),ELAST(3,3)
C      COMMON /INLAM/ LAMAT(50,100),LYRND(50),INDK(10)
C      COMMON /FLAM/ TKLAM(50,100),ORLAM(50,100)
C      DIMENSION FVEC(24), CLOAD(217,2)
C      PI = 3.14159265359D0
C      MEL = 182
C      MJT = 217
C      MMT = 10
C      MBD = 86
C      MLEL = 50
C      MLYR = 100
C      NZRO = 0
C      ZRO = 0.0D0
C      1 CALL INPUT
C
C      DO 2 I=1,MJT
C
C      DC 2 J=1,2
C      2 CLCAD(I,J) = ZRO
C
C      NA = NSTRES
C      NEG = 2*NJT
C
C      DO 15 I=1,NEL
C      NPEL = NCON(I,15)*4
C      NJPE = NPEL*2
C      INDIC = NCON(I,15)
C      IF (INDIC.NE.1) GO TO 3
C      CALL LAMC (I)
C      GC TO 5

```

```

3  IMAT = NCON(I,I4)
   THETA = ANGL(I)
   INCLC = INDK(IMAT)
   CALL STFMAT(IMAT,THETA,INDIC,NSTRES)
   IF (INDIC.NE.0) GO TO 4
   ALPHA = ELCON(IMAT,6)
   IF (NN.NE.0) ALPHA = ALPHA*(1.000+ELCON(IMAT,2))
4  CCNTINUE
   TK = ELCON(IMAT,7)
   IF (NSTRES.EQ.1) TK=1.000
5  CCNTINUE

C
   DO 6 J=1,NPEL
   IJ = NCCN(I,J)
   LM(J) = IJ
   TEMP(J) = THNP(IJ)

C
   DO 6 JJ=1,2
   BCFY(J,JJ) = BYNP(IJ,JJ)
   DSPN(J,JJ) = DPIN(IJ,JJ)
6  COORD(J,JJ) = COARD(IJ,JJ)

C
   IF (NTLD.EQ.0) GO TO 9
   LCODE = I
   CALL FLOAD (FVEC,NJPE,LCODE,NGP)
   WRITE (6,23) I,NZRO,NZRO

C
   DO 7 L=1,NPEL
   IO = 2*L-1
   IE = IO+1
   J = NCCN(I,L)
7  WRITE (6,24) J,FVEC(IO),FVEC(IE)

C
C
   DO 8 J=1,NPEL
   IJ = LM(J)
   IO = 2*J-1
   IE = IO+1
   CLOAD(IJ,1) = FVEC(IO)+CLOAD(IJ,1)
   CLOAD(IJ,2) = FVEC(IE)+CLOAD(IJ,2)
8

C
   IF (NTBY.EQ.0) GO TO 12
   LCODE = 2
   CALL FLOAD (FVEC,NJPE,LCODE,NGP)
   WRITE (6,23) I,NZRO,NTBY,NZRO

C
   DO 10 L=1,NPEL
   IO = 2*L-1

```

```

MAIN1890
MAIN1900
MAIN1910
MAIN1920
MAIN1930
MAIN1940
MAIN1950
MAIN1960
MAIN1970
MAIN1980
MAIN1990
MAIN2000
MAIN2010
MAIN2020
MAIN2030
MAIN2040
MAIN2050
MAIN2060
MAIN2070
MAIN2080
MAIN2090
MAIN2100
MAIN2110
MAIN2120
MAIN2130
MAIN2140
MAIN2150
MAIN2160
MAIN2170
MAIN2180
MAIN2190
MAIN2200
MAIN2210
MAIN2220
MAIN2230
MAIN2240
MAIN2250
MAIN2260
MAIN2270
MAIN2280
MAIN2290
MAIN2300
MAIN2310
MAIN2320
MAIN2330
MAIN2340
MAIN2350
MAIN2360

```



```

      IE = IO+1
      J = NCON(I,L)
10  WRITE (6,24) J,FVEC(IO),FVEC(IE)
C
C
      DO 11 J=1,NPEL
      IJ = LM(J)
      IO = 2*J-1
      IE = IO+1
11  CLOAD(IJ,1) = FVEC(IO)+CLOAD(IJ,1)
      CLOAD(IJ,2) = FVEC(IE)+CLOAD(IJ,2)
C
12  IF (NTIN.EQ.0) GO TO 15
      LCODE = 3
      CALL FLOAD (FVEC,NJPE,LCODE,NGP)
      WRITE (6,23) I,NZRO,NZRO,NTIN
C
      DO 13 L=1,NPEL
      IC = 2*L-1
      IE = IO+1
      J = NCON(I,L)
13  WRITE (6,24) J,FVEC(IO),FVEC(IE)
C
C
      DO 14 J=1,NPEL
      IJ = LM(J)
      IO = 2*J-1
      IE = IO+1
      CLOAD(IJ,1) = FVEC(IO)+CLOAD(IJ,1)
      CLOAD(IJ,2) = FVEC(IE)+CLOAD(IJ,2)
14  CLOAD(IJ,2) = FVEC(IE)+CLOAD(IJ,2)
C
15  CCNTINUE
C
      KJT = NJT
C
      DC 19 I=1,NPBC
      NCODE = NBC(I,2)
      NJT = NBC(I,1)
      IF ((NCODE.EQ.1).OR.(NCODE.EQ.4)) CLOAD(NJT,2)=ZRO
      IF ((NCODE.EQ.2).OR.(NCODE.EQ.5)) CLOAD(NJT,1)=ZRO
      IF ((NCODE.EQ.3).OR.(NCODE.EQ.6)) GO TO 16
      GO TO 17
16  CLOAD(NJT,1) = ZRO
      CLCAD(NJT,2) = ZRO
17  IF (NCODE.EQ.7) GO TO 18
      GO TO 19
18  ALF = TLOAD(NJT,2)*PI/180.0D0
      COSA = DCOS(ALF)

```

```

MA IN2370
MA IN2380
MA IN2390
MA IN2400
MA IN2410
MA IN2420
MA IN2430
MA IN2440
MA IN2450
MA IN2460
MA IN2470
MA IN2480
MA IN2490
MA IN2500
MA IN2510
MA IN2520
MA IN2530
MA IN2540
MA IN2550
MA IN2560
MA IN2570
MA IN2580
MA IN2590
MA IN2600
MA IN2610
MA IN2620
MA IN2630
MA IN2640
MA IN2650
MA IN2660
MA IN2670
MA IN2680
MA IN2690
MA IN2700
MA IN2710
MA IN2720
MA IN2730
MA IN2740
MA IN2750
MA IN2760
MA IN2770
MA IN2780
MA IN2790
MA IN2800
MA IN2810
MA IN2820
MA IN2830
MA IN2840

```

```

C      SINA = DSIN(ALF)
C      CLCAD(NJT,1) = CLOAD(NJT,1)*COSA+CLOAD(NJT,2)*SINA
C      CLOAD(NJT,2) = ZRO
15  CCNTINUE
C
C      NJT = KJT
C      WRITE (6,22)
C
C      DC 20 I=1,NJT
C      IF (NCARD.NE.0) WRITE (7,21) I,CLOAD(I,1),CLOAD(I,2),THNP(I)
C      WRITE (6,21) I,CLOAD(I,1),CLOAD(I,2),THNP(I)
20  CCNTINUE
C
C      GC TO 1
C
21  FORMAT (5X,115,2G25.16,1G15.8)
22  FORMAT (//,10X,' RESULTANT LOAD VECTOR (NTLD+NTBY+NTIN)',/,
1  4X,' JOINT X COMPONENT',12X,' Y COMPONENT',10X,'NODAL TEM
2  P',//)
23  FORMAT (//,5X,'ELEMENT',113,' LOAD VECTOR',313,/,5X,'JOINT',7X,'
1FX',13X,'FY',//)
24  FORMAT (5X,115,2G15.6)
END
SUBROUTINE INPUT
IMPLICIT REAL*8(A-H,O-Z)
IMPLICIT INTEGER*2(I-N)
COMMON /INT/ NEL,NJT,NMAT,NCLOAD,NPBC,NCON(182,15),NBC(217,2),NSTR
1ES,NGP,LM(12),LJT(12),NLEL
COMMON /MDIM/ MEL,MJT,MMT,MBD,MLEL,MLYR
COMMON /FLPL/ COORD(217,2),CLOAD(217,2),ELCON(10,7),TITLE(10),THNP
1(217),BYNP(217,2),DPIN(217,2),ANGL(182)
COMMON /LOAD/ TEMP(12),BDFY(12,2),DSPN(12,2),TK,ALPHA,NN,NTLD,NTBY
1,NTIN,NCARD
COMMON /INLAM/ LAMAT(50,100),LYRNO(50),INDK(10)
COMMON /FLLAM/ TKLAM(50,100),ORLAM(50,100)
DATA CHK/,STOP
PI = 3.14159265359D0
ZRC = 0.0D0
READ (5,25) TITLE
WRITE (6,26) TITLE
IF (TITLE(1).EQ.CHK) STOP
C
C      DC 1 I=1,MJT
C      THNP(I) = ZRO
C
C      DC 1 J=1,2
C      COORD(I,J) = ZRO
C      CLCAD(I,J) = ZRO

```



5	CONTINUE	INPT	740
C		INPT	750
C		INPT	760
6	DO 6 I=1,NJT WRITE (6,31) I,COORD(I,1),COORD(I,2),CLOAD(I,1),CLOAD(I,2)	INPT	770
C		INPT	780
C		INPT	790
C		INPT	800
C		INPT	810
C		INPT	820
C		INPT	830
7	DO 7 I=1,NMAT READ (5,32) IMAT,INDK(IMAT),(ELCON(IMAT,J),J=1,7)	INPT	840
C		INPT	850
C		INPT	860
	WRITE (6,33)	INPT	870
8	DO 8 I=1,NMAT WRITE (6,34) I,(ELCON(I,J),J=1,7),INDK(I)	INPT	880
C		INPT	890
C		INPT	900
C		INPT	910
C		INPT	920
C		INPT	930
C		INPT	940
C		INPT	950
C		INPT	960
C		INPT	970
9	DO 9 I=1,NEL READ (5,37) IJT,(NCON(IJT,J),J=1,14),ANGL(IJT),NCON(IJT,15)	INPT	980
C		INPT	990
C		INPT	1000
C		INPT	1010
C		INPT	1020
C		INPT	1030
C		INPT	1040
C		INPT	1050
C		INPT	1060
C		INPT	1070
C		INPT	1080
C		INPT	1090
C		INPT	1100
10	DO 10 I=1,NEL WRITE (6,38) I,(NCON(I,J),J=1,14),ANGL(I),NCON(I,15)	INPT	1110
C		INPT	1120
C		INPT	1130
C		INPT	1140
C		INPT	1150
C		INPT	1160
C		INPT	1170
C		INPT	1180
C		INPT	1190
C		INPT	1200
C		INPT	1210

```

NBC(I,1) = NBC(J,1)
NBC(I,2) = NBC(J,2)
NBC(J,1) = NT1
NBC(J,2) = NT2
12 CCNTINUE
C
NSV = 0
C
DO 13 I=1,NPBC
IF (NBC(I,2).NE.7) GO TO 13
NSV = NSV+1
13 CCNTINUE
C
APBCM = NPBC-1
IF (NSV.EQ.0) GO TO 18
C
DC 16 I=1,NSV
C
DO 15 J=1,NPBCM
IF (NBC(J,2).NE.7) GO TO 15
NT1 = NBC(J,1)
C
DO 14 K=J,NPBCM
KP = K+1
NBC(K,1) = NBC(KP,1)
NBC(K,2) = NBC(KP,2)
14
C
NBC(NPBC,1) = NT1
NBC(NPBC,2) = 7
GO TO 16
15 CCNTINUE
C
16 CCNTINUE
C
WRITE (6,39)
C
DC 17 I=1,NPBC
17 WRITE (6,41) (NBC(I,J),J=1,2)
C
18 CCNTINUE
IF (NLEL.EQ.0) GO TO 21
C
READ LAMINATE CARD DECK
C
C
DO 19 I=1,NLEL
READ (5,42) LAMTP,LYRNO(LAMTP)
L = LYRNO(LAMTP)
19

```



```

2, 'THICKNESS', 2X, 'INDK')
34 FORMAT (, , 1X, I3, 5X, 7(2X, G13.6), 2X, I5)
35 FORMAT (, , , ELEMENT CARDS - CONNECTIVITY MATRIX, )
36 FORMAT (, , , ELEMENT NUMBER, 20X, 'JOINT NUMBERS', 30X, 'KIND', 3X, 'MA
1 TERIAL', 3X, 'THETA(DEG.)', 3X, 'LAM. IND.')
37 FCRMAT (15I4, F10.5, I5)
38 FCRMAT (6X, I3, 6X, I2I5, 1X, I5, 5X, I5, 1X, F10.5, 5X, I5)
39 FCRMAT (, , , BOUNDARY CONDITION CODE, //, 5X, 'JNT NO', 6X, 'CODE', //)
40 FCRMAT (2I5)
41 FCRMAT (5X, I5, I10)
42 FCRMAT (2I5)
43 FCRMAT (4I3, F8.3, F9.4)
44 FCRMAT (, , , LAMINATE PROPERTIES, //, 1X, 'LAMINATE TYPE', 5X, 'NO OF
1 LAYERS', 1X, 'LAYER NUMBER', 5X, 'MATERIAL NO.', 5X, 'THICKNESS', 5X,
2 ORIENTATION, //)
45 FCRMAT (4X, I5, 11X, I5)
46 FCRMAT (30X, I5, 8X, I7, 10X, F9.3, 5X, F9.3)
47 FCRMAT (, , , DISTRIBUTED LOAD INFORMATION, //, , , INITIAL DISP., /, 2X,
1 , JOINT TEMP. X COMP. Y COMP. Y COMP., //)
2 FCRMAT (115, 5F15.0)
48 FCRMAT (4X, I13, 3X, 5G10.3)
49 FCRMAT (, , , 5X, , *** NO DISTRIBUTED LOAD *** )
50 FORMAT (, , , 5X, , *** NO DISTRIBUTED LOAD *** )
END
SUBROUTINE LAMC (IK)
THIS SUBROUTINE CALCULATES THE STIFFNESS MATRIX FOR LAMINATED
COMPOSITE MATERIALS. THIS SUBROUTINE ASSUMES THAT THE LAMINATED
COMPOSITE IS OF BALANCED DESIGN SO THAT THE STIFFNESS AND BENDING
PROPERTIES ARE UNCOUPLED. IT CALCULATES AN AVERAGE STIFFNESS MATRIX
FROM THE STIFFNESS PROPERTIES OF EACH LAMINA.
IK IS THE ELEMENT NUMBER
IMPLICIT REAL*8(A-H, O-Z)
IMPLICIT INTEGER*2(I-N)
COMMON /FLPL/ COARD(217,2), TLOAD(217,2), ELCON(10,7), TITLE(10), THNP
1 (217), BYNP(217,2), DPIN(217,2), ANGL(182)
COMMON /INT/ NEL, NJT, NMAT, NLOAD, NPBC, NCON(182,15), NBC(217,2), NSTR
1 ES, NGP, LM(12), LJT(12), NLEL
COMMON /LOAD/ TEMP(12), BDFY(12,2), DSPN(12,2), TK, ALPHA, NN, NTLD, NTBY
1 NTIN, NCARD
COMMON /INLAM/ LAMAT(50,100), LYRNO(50), INDK(10)
COMMON /FLLAM/ TKLAM(50,100), ORLAM(50,100)
COMMON /CC/ COORD(12,2), ELAST(3,3)
DIMENSION C(3,3)
LAMTP = NCON(IK, 14)
THETA = ANGL(IK)
TK = 0.00

```

DC 1 I=1,3

C

C C C C C C



```

C      DC 1 J=1,3
C      1 C(I,J) = 0.D0
C
C      LYRS = LYRNO(LAMTP)
C
C      DO 3 I=1,LYRS
C      MATN = LAMAT(LAMTP,I)
C      PHI = THETA+ORLAM(LAMTP,I)
C      INDIC = INDK(MATN)
C      NSTRES = NN
C      CALL STFMAT (MATN,PHI,INDIC,NSTRES)
C      TK = TK+TKLAM(LAMTP,I)
C
C      DC 2 J=1,3
C
C      DC 2 K=1,3
C      2 C(J,K) = C(J,K)+ELAST(J,K)*TKLAM(LAMTP,I)
C      3 CCNTINUE
C
C      DO 4 J=1,3
C
C      DC 4 K=1,3
C      4 ELAST(J,K) = C(J,K)/TK
C
C      RETURN
C      END
C      SUBROUTINE STFMAT (MATN,THETA,INDIC,STIFFNESS, MATRIX FOR A HOOKEAN
C      THIS SUBROUTINE ASSEMBLES THE ELASTIC STIFFNESS IN C(3X3) THE VALUES
C      ELEMENT. THE SUBROUTINE CALCULATES AND STORES IN C(3X3) THE VALUES
C      OF THE ELEMENTS OF THIS MATRIX FOR ISOTROPIC MATERIALS UNDER
C      EITHER PLANE STRAIN OR PLANE STRESS OR FOR ANISOTROPIC MATERIALS
C      EXHIBITING TWO DIMENSIONAL ORTHOTROPY. FOR ANISOTROPIC MATERIALS,
C      THE ELEMENTS OF THIS MATRIX ARE FIRST COMPUTED IN THE MATERIAL
C      BASED L-T COORDINATE SYSTEM AND THEN TRANSFORMED TO THE PROBLEM
C      BASED X-Y COORDINATE SYSTEM.
C      IMPLICIT REAL*8(A-H,O-Z)
C      INTEGER*2(I-N)
C      COMMON /FLPL/ COORD(217,2), TLOAD(217,2), ELCON(10,7), TITLE(10), THNPS
C      1(217), BYNP(217,2), DPIN(217,2), ANGL(182)
C      COMMON /CC/ COORD(12,2), C(3,3)
C      DIMENSION A(3,3)
C
C      DC 1 I=1,3
C
C      DC 1 J=1,3

```

LAMC 250  
 LAMC 260  
 LAMC 270  
 LAMC 280  
 LAMC 290  
 LAMC 300  
 LAMC 310  
 LAMC 320  
 LAMC 330  
 LAMC 340  
 LAMC 350  
 LAMC 360  
 LAMC 370  
 LAMC 380  
 LAMC 390  
 LAMC 400  
 LAMC 410  
 LAMC 420  
 LAMC 430  
 LAMC 440  
 LAMC 450  
 LAMC 460  
 LAMC 470  
 LAMC 480  
 LAMC 490  
 LAMC 500  
 LAMC 510  
 LAMC 520  
 LAMC 530  
 STFM 10  
 STFM 20  
 STFM 30  
 STFM 40  
 STFM 50  
 STFM 60  
 STFM 70  
 STFM 80  
 STFM 90  
 STFM 100  
 STFM 110  
 STFM 120  
 STFM 130  
 STFM 140  
 STFM 150  
 STFM 160  
 STFM 170  
 STFM 180  
 STFM 190



```

C      A(I,J) = 0.000
C      C(I,J) = 0.000
C      1 CONTINUE
C
C      IF (INDIC.EQ.1) GO TO 3
C      IF (NSTRES.EQ.1) GO TO 2
C      FORM THE ELASTIC STIFFNESS MATRIX FOR ISOTROPIC PLANE STRESS
C
C      E = ELCCN(MATN,1)
C      XNU = ELCON(MATN,2)
C      EO = E/(1.000-XNU*XNU)
C      C(1,1) = EO
C      C(1,2) = EO*XNU
C      C(2,1) = C(1,2)
C      C(2,2) = C(1,1)
C      C(3,3) = EO*(1.000-XNU)/2.000
C      RETURN
C      2 CCNTINUE
C
C      FORM THE ELASTIC STIFFNESS MATRIX FOR ISOTROPIC PLANE STRAIN
C
C      E = ELCCN(MATN,1)
C      XNU = ELCON(MATN,2)
C      EI = E/((1.000+XNU)*(1.000-2.000*XNU))
C      C(1,1) = EI*(1.000-XNU)
C      C(1,2) = EI*XNU
C      C(2,1) = C(1,2)
C      C(2,2) = C(1,1)
C      C(3,3) = EI*(0.500-XNU)
C      RETURN
C      3 CCNTINUE
C
C      FORM THE ANISOTROPIC ORTHOTROPIC ELASTIC MATRIX IN THE MATERIAL BASED
C      L-T COORDINATE SYSTEM
C
C      EL = ELCON(MATN,1)
C      ET = ELCON(MATN,2)
C      XNULT = ELCON(MATN,3)
C      XNUTL = ELCON(MATN,4)
C      GLT = ELCON(MATN,5)
C      IF (EL.LT.-0.000) EL=XNULT*ET/XNUTL
C      IF (ET.LT.-0.000) ET=EL*XNUTL/XNULT
C      IF (XNULT.LT.-0.000) XNULT=EL*XNUTL/ET
C      IF (XNUTL.LT.-0.000) XNUTL=XNULT*ET/EL
C      Z = 1.000-XNULT*XNUTL
C      A(1,1) = EL/Z
C      A(1,2) = ET*XNULT/Z
C      A(2,1) = A(1,2)

```

200  
 STFM 210  
 STFM 220  
 STFM 230  
 STFM 240  
 STFM 250  
 STFM 260  
 STFM 270  
 STFM 280  
 STFM 290  
 STFM 300  
 STFM 310  
 STFM 320  
 STFM 330  
 STFM 340  
 STFM 350  
 STFM 360  
 STFM 370  
 STFM 380  
 STFM 390  
 STFM 400  
 STFM 410  
 STFM 420  
 STFM 430  
 STFM 440  
 STFM 450  
 STFM 460  
 STFM 470  
 STFM 480  
 STFM 490  
 STFM 500  
 STFM 510  
 STFM 520  
 STFM 530  
 STFM 540  
 STFM 550  
 STFM 560  
 STFM 570  
 STFM 580  
 STFM 590  
 STFM 600  
 STFM 610  
 STFM 620  
 STFM 630  
 STFM 640  
 STFM 650  
 STFM 660  
 STFM 670

```

C      A(2,2) = ET/Z
C      A(3,3) = GLT
C
C      THIS ARRAY IS NOW TRANSFORMED FROM THE MATERIAL L-T COORDINATE
C      SYSTEM TO THE PROBLEM X-Y COORDINATE SYSTEM. THE TRANSFORMATION IS
C      ACCOMPLISHED BY ROTATION ABOUT THE Z AXIS THROUGH THETA DEGREES.
C
      PI = 3.14159265359D0
      THRAD = THETA*PI/180.0D0
      CCS1 = DCOS(THRAD)**2
      CCS2 = DCOS(THRAD)**3
      CCS3 = DCOS(THRAD)**4
      CCS4 = DCOS(THRAD)**4
      SIN1 = DSIN(THRAD)**2
      SIN2 = DSIN(THRAD)**2
      SIN3 = DSIN(THRAD)**3
      SIN4 = DSIN(THRAD)**4
      TWO = 2.0D0
      FOUR = 4.0D0
      C(1,1) = A(1,1)*COS4+TWO*(A(1,2)+TWO*A(3,3))*SIN2*COS2+A(2,2)*SIN4
      C(1,2) = (A(1,1)+A(2,2))-FOUR*A(3,3))*SIN2*COS2+A(1,2)*(SIN4+COS4)
      C(1,3) = (A(1,1)-A(1,2)-TWO*A(3,3))*SIN1*COS3+(A(1,2)-A(2,2)+TWO*
1A(3,3))*SIN3*COS1
      C(2,2) = A(1,1)*SIN4+TWO*(A(1,2)+TWO*A(3,3))*SIN2*COS2+A(2,2)*COS4
      C(2,3) = (A(1,1)-A(1,2)-TWO*A(3,3))*SIN3*COS1+(A(1,2)-A(2,2)+TWO*
1C(3,3))*SIN1*COS3
      C(3,3) = (A(1,1)+A(2,2)-TWO*(A(1,2)+A(3,3))*SIN2*COS2+A(3,3)*(SIN
14+COS4)
      C(2,1) = C(1,2)
      C(3,1) = C(1,3)
      C(3,2) = C(2,3)
      RETURN
      ENC
      SUBROUTINE FORMB (B,X,Y,N,DTJ,IND)
C
C      THIS SUBROUTINE FORMS THE B MATRIX AS A FUNCTION OF
C      XI AND ETA FOR THE THREE DIFFERENT QUADRILATERAL ELEM. IN THIS FAMILY
C
      IMPLICIT REAL*8(A-H,O-Z)
      IMPLICIT INTEGER*2(I-N)
      DIMENSION AJ(2,2), AJIN(2,2), DNX(2,12), W1(2,12), B(3,24)
      COMMON /CC/ COORD(12,2),ELAST(3,3)
      ZERO = 0.0D0
C
      DO 1 I=1,3
C
      DO 1 J=1,N
1 B(I,J) = ZERO

```

```
C      ONE = 1.0D0
C      TWO = 2.0D0
C      FCUR = 4.0D0
C      NPT = N/2
C
C FORM (2XNPT) MATRIX OF DERIV. OF THE INTERP. FUNCT. WRT XI AND ETA
C
C      IGO = N/8
C      GO TO (2,3,4), IGO
C
C      LINEAR FUNCTIONS
C
C      2 W1(1,3) = -(ONE-Y)/FOUR
C        W1(1,4) = -W1(1,3)
C        W1(1,1) = (ONE+Y)/FOUR
C        W1(1,2) = -W1(1,1)
C        W1(2,3) = -(ONE-X)/FOUR
C        W1(2,4) = -(ONE+X)/FOUR
C        W1(2,1) = -W1(2,4)
C        W1(2,2) = -W1(2,3)
C        GO TO 5
C      3 CCNTINUE
C
C      QUADRATIC FUNCTIONS
C
C      TXPY = TWO*X+Y
C      TXMY = TWO*X-Y
C      TYPX = TWO*Y+X
C      TYMX = TWO*Y-X
C      OMY = ONE-Y
C      OPY = ONE+Y
C      OPX = ONE+X
C      OMX = ONE-X
C      W1(1,5) = OMY*TXPY/FOUR
C      W1(1,6) = -OMY*X
C      W1(1,7) = OMY*TXMY/FOUR
C      W1(1,8) = OPY*QMY/TWO
C      W1(1,1) = OPY*TXPY/FOUR
C      W1(1,2) = -OPY*X
C      W1(1,3) = OPY*TXMY/FOUR
C      W1(1,4) = -OPY*OMY/TWO
C      W1(2,5) = OMX*TYPX/FOUR
C      W1(2,6) = -OPX*OMX/TWO
C      W1(2,7) = OPX*TYMX/FOUR
C      W1(2,8) = -Y*OPX
C      W1(2,1) = OPX*TYPX/FOUR
C      W1(2,2) = OPX*QMX/TWO
```



```

      W1(2,6) = -OMX*TPMTMNY/TTN
      5 CCNTINUE
      C
      C FORM JACOBIAN OF THE TRANSFORMATION IN AJ(I,J)
      C
      C
      C DO 6 I=1,2
      C
      C DO 6 J=1,2
      C AJ(I,J) = ZERO
      C
      C DC 6 K=1,NPT
      C 6 AJ(I,J) = AJ(I,J)+W1(I,K)*COORD(K,J)
      C
      C CALCULATE DETERMINANT OF THE JACOBIAN
      C
      C DTJ = AJ(1,1)*AJ(2,2)-AJ(1,2)*AJ(2,1)
      C IF (IND.NE.0) RETURN
      C
      C INVERT JACOBIAN IN AJIN
      C
      C AJIN(1,1) = AJ(2,2)/DTJ
      C AJIN(1,2) = -AJ(1,2)/DTJ
      C AJIN(2,1) = -AJ(2,1)/DTJ
      C AJIN(2,2) = AJ(1,1)/DTJ
      C
      C FORM (2XNPT) MATRIX OF DERIVATIVES OF INTERP. FUNCT WRT X AND Y
      C
      C DC 7 I=1,2
      C
      C DC 7 J=1,NPT
      C DNX(I,J) = ZERO
      C
      C DC 7 K=1,2
      C 7 DNX(I,J) = DNX(I,J)+AJIN(I,K)*W1(K,J)
      C
      C FORM B(I,J) MATRIX
      C
      C DC 8 I=1,NPT
      C ICD = 2*I-1
      C IEV = 2*I
      C B(1,ICD) = DNX(1,I)
      C B(2,IEV) = DNX(2,I)
      C B(3,IEV) = DNX(3,I)
      C

```

```

FRMB11120
FRMB11130
FRMB11140
FRMB11150
FRMB11160
FRMB11170
FRMB11180
FRMB11190
FRMB11200
FRMB11210
FRMB11220
FRMB11230
FRMB11240
FRMB11250
FRMB11260
FRMB11270
FRMB11280
FRMB11290
FRMB11300
FRMB11310
FRMB11320
FRMB11330
FRMB11340
FRMB11350
FRMB11360
FRMB11370
FRMB11380
FRMB11390
FRMB11400
FRMB11410
FRMB11420
FRMB11430
FRMB11440
FRMB11450
FRMB11460
FRMB11470
FRMB11480
FRMB11490
FRMB11500
FRMB11510
FRMB11520
FRMB11530
FRMB11540
FRMB11550
FRMB11560
FRMB11570
FRMB11580
FRMB11590

```





```

18451374D0/
DATA X5/.9061798459386D0,.5384693101056D0,0.0D0,-.5384693101056D0,
1-.9061798459386D0/
LCATA A5/.2369268850561D0,.4786286704993D0,.5688888888888D0,.478628
16704993D0,.2369268850561D0/
IF (NGP.EQ.0) NGP = 5
IF (TK.EQ.0.0D0) TK=1.0D0
C
DC 1 I=1,24
FVEC(I) = 0.0D0
1 WVEC(I) = 0.0D0
C
JUMP = NGP-1
C
DC 6 I=1,NGP
GO TO (2,3,4,5), JUMP
2 XI(I) = X2(I)
AI(I) = A2(I)
GO TO 6
3 XI(I) = X3(I)
AI(I) = A3(I)
GO TO 6
4 XI(I) = X4(I)
AI(I) = A4(I)
GO TO 6
5 XI(I) = X5(I)
AI(I) = A5(I)
6 CCNTINUE
C
DC 7 I=1,NGP
DO 7 J=1,NGP
7 AIA(I,J) = AI(I)*AI(J)
NPEL = N/2
C
DC 24 I=1,NGP
X = XI(I)
C
DO 24 J=1,NGP
Y = XI(J)
GC TO (8,13,17), LCODE
8 ILT = 0
CALL FORMB (B,X,Y,N,DIJ,ILT)
C
DO 9 II=1,N
C

```

```

C      DO 9 JJ=1,3
      BTD(II,JJ) = 0.000
      FL0D 930

C      DO 9 KK=1,3
      BTD(II,JJ) = BTD(II,JJ)+B(KK,II)*ELAST(KK,JJ)
      FL0D 940

C      CALL SHAPE (VAL,X,Y,NPEL)
      CFT = 0.000
      FL0D 950

C      DC 10 II=1,NPEL
      CFT = CFT+VAL(II)*TEMP(II)
      FL0D 960

C      CFT = CFT*ALPHA*TK*DTJ
      FL0D 970

C      DO 11 II=1,N
      WVEC(II) = (BTD(II,1)+BTD(II,2))*CFT
      FL0D 980

C      DC 12 II=1,N
      FVEC(II) = FVEC(II)+WVEC(II)*AIA(I,J)
      FL0D 990

C      GC TO 24
      CALL SHAPE (VAL,X,Y,NPEL)
      FL0D 1000

C      ILT = 1
      CALL FORMB (B,X,Y,N,DTJ,ILT)
      FL0D 1010

C      CFTX = 0.000
      CFTY = 0.000
      FL0D 1020

C      DC 14 II=1,NPEL
      CFTX = CFTX+VAL(II)*BDFY(II,1)
      FL0D 1030

C      CFTY = CFTY+VAL(II)*BDFY(II,2)
      FL0D 1040

C      DC 15 II=1,NPEL
      IC = 2*II-1
      IE = IO+1
      FL0D 1050

C      WVEC(IE) = VAL(II)*CFTX*DTJ*TK
      WVEC(IE) = VAL(II)*CFTY*DTJ*TK
      FL0D 1060

C      DC 16 II=1,N
      FVEC(II) = FVEC(II)+WVEC(II)*AIA(I,J)
      FL0D 1070

C      GC TO 24
      ILT = 0
      FL0D 1080

C      CALL FORMB (B,X,Y,N,DTJ,ILT)
      FL0D 1090

C      DO 18 II=1,N
      FL0D 1100
      FL0D 1110
      FL0D 1120
      FL0D 1130
      FL0D 1140
      FL0D 1150
      FL0D 1160
      FL0D 1170
      FL0D 1180
      FL0D 1190
      FL0D 1200
      FL0D 1210
      FL0D 1220
      FL0D 1230
      FL0D 1240
      FL0D 1250
      FL0D 1260
      FL0D 1270
      FL0D 1280
      FL0D 1290
      FL0D 1300
      FL0D 1310
      FL0D 1320
      FL0D 1330
      FL0D 1340
      FL0D 1350
      FL0D 1360
      FL0D 1370
      FL0D 1380
      FL0D 1390
      FL0D 1400

```



```

C      DO 18 JJ=1,3
C      BTD(II,JJ) = 0.0D0
C
C      DO 18 KK=1,3
C      18 BTD(II,JJ) = BTD(II,JJ)+B(KK,II)*ELAST(KK,JJ)
C
C      DO 19 II=1,N
C      DO 19 JJ=1,N
C      SK(II,JJ) = 0.0D0
C
C      DO 19 KK=1,3
C      19 SK(II,JJ) = SK(II,JJ)+BTD(II,KK)*B(KK,JJ)
C
C      DC 20 II=1,NPEL
C      IC = 2*II-1
C      IF = IO+1
C      DSFO(IO) = DSPN(II,1)
C      DSPO(IE) = DSPN(II,2)
C
C      DO 21 II=1,N
C      WVEC(II) = 0.0D0
C
C      DC 21 JJ=1,N
C      21 WVEC(II) = WVEC(II)+SK(II,JJ)*DSPC(JJ)
C
C      DC 22 II=1,N
C      22 WVEC(II) = -WVEC(II)*TK*DTJ
C
C      DC 23 II=1,N
C      23 FVEC(II) = FVEC(II)+WVEC(II)*AIA(I,J)
C      24 CCNTINUE
C      RETURN
C      END
C      SCBRROUTINE SHAPE (VAL,X,Y,NPEL)
C      IMPLICIT REAL*8(A-H,O-Z)
C      IMPLICIT INTEGER*2(I-N)
C      DIMENSION VAL(12), XYL(4,2), XYQ(8,2), XYC(12,2), IPERM(4)
C      DATA XYL/1.0D0,-1.0D0,-1.0D0,-1.0D0,-1.0D0,-1.0D0,-1.0D0,-1.0D0/,
C      DATA XYQ/1.0D0,0.0D0,-1.0D0,-1.0D0,-1.0D0,-1.0D0,0.0D0,1.0D0,1.0D0,1.0D0,
C      FL0D1410
C      FL0D1420
C      FL0D1430
C      FL0D1440
C      FL0D1450
C      FL0D1460
C      FL0D1470
C      FL0D1480
C      FL0D1490
C      FL0D1500
C      FL0D1510
C      FL0D1520
C      FL0D1530
C      FL0D1540
C      FL0D1550
C      FL0D1560
C      FL0D1570
C      FL0D1580
C      FL0D1590
C      FL0D1600
C      FL0D1610
C      FL0D1620
C      FL0D1630
C      FL0D1640
C      FL0D1650
C      FL0D1660
C      FL0D1670
C      FL0D1680
C      FL0D1690
C      FL0D1700
C      FL0D1710
C      FL0D1720
C      FL0D1730
C      FL0D1740
C      FL0D1750
C      FL0D1760
C      FL0D1770
C      FL0D1780
C      FL0D1790
C      FL0D1800
C      FL0D1810
C      FL0D1820
C      SHPE 10
C      SHPE 20
C      SHPE 30
C      SHPE 40
C      SHPE 50
C      SHPE 60

```

[illegible]

```

      C      Y1 = XVC(I,2)
      8 VAL(I) = FCC(X,Y,X1,Y1)
      C      IPERM(1) = 2
      IPERM(2) = 3
      IPERM(3) = 8
      IPERM(4) = 9
      C      DO 9 I=1,4
      IJ = IPERM(I)
      X1 = XVC(IJ,1)
      Y1 = XVC(IJ,2)
      9 VAL(IJ) = FCM(X,Y,X1,Y1)
      C      IPERM(1) = 5
      IPERM(2) = 6
      IPERM(3) = 11
      IPERM(4) = 12
      C      DO 10 I=1,4
      IJ = IPERM(I)
      X1 = XVC(IJ,1)
      Y1 = XVC(IJ,2)
      10 VAL(IJ) = FCM(Y,X,Y1,X1)
      C      RETURN
      END

```

```

SHPE 550
SHPE 560
SHPE 570
SHPE 580
SHPE 590
SHPE 600
SHPE 610
SHPE 620
SHPE 630
SHPE 640
SHPE 650
SHPE 660
SHPE 670
SHPE 680
SHPE 690
SHPE 700
SHPE 710
SHPE 720
SHPE 730
SHPE 740
SHPE 750
SHPE 760
SHPE 770
SHPE 780
SHPE 790
SHPE 800
SHPE 810

```

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